Integration Techniques

Problem Set 23

1. Find the integral of $\displaystyle\int \frac{x}{\sqrt{1+x^2}}\ dx$ by first making the substitution $u = 1 + x^2$.

2. Evaluate the following integrals by using the method of substitution.

   (a) $\displaystyle\int x^2(x^3-2)^5\ dx$

   (b) $\displaystyle\int x^2 e^{x^3}\ dx$

   (c) $\displaystyle\int x^4\cos(x^5+3)\ dx$

   (d) $\displaystyle\int \frac{x}{x^2+1}\ dx$

   (e) $\displaystyle\int \frac{2x}{x^2+4}\ dx$

   (f) $\displaystyle\int \frac{3x^2}{5+x^3}\ dx$

   (g) $\displaystyle\int \frac{2x+1}{x^2+x-3}\ dx$

   (h) $\displaystyle\int \frac{6x^2+8x}{x^3+2x^2+7}\ dx$

   (i) $\displaystyle\int \frac{(\log x)^2}{x}\ dx$

   (j) $\displaystyle\int \sin(3x+2)\ dx$

3. Evaluate the following definite integrals.

   (a) $\displaystyle\int_0^2 \frac{3x}{x^2+1}\ dx$

   (b) $\displaystyle\int_0^1 x^2(1-2x^3)^3\ dx$

   (c) $\displaystyle\int_0^4 \frac{\sin\sqrt{x}}{\sqrt{x}}\ dx$

4. Find the following integrals.

   (a) $\displaystyle\int e^{\sin x}\cos x\ dx$

   (b) $\displaystyle\int \tan x\ dx$

   (c) $\displaystyle\int \sin x\cos x\ dx$

   (d) $\displaystyle\int \cos x\sin^3 x\ dx$

   (e) $\displaystyle\int \sin^3 x\ dx$

   (f) $\displaystyle\int \frac{\cos x}{(1+\sin x)^2}\ dx$

5. Use the Left-end, Right-end and Midpoint rules to find $\displaystyle\int_0^2 x^2\ dx$ using 4 sub-intervals.

6. Use the Midpoint Rule to approximate $\displaystyle\int_0^\pi \sqrt{\sin x}\ dx$ using 3 sub-intervals.

7. The velocity of an insect is given by $v(t) = \ln(t^2+1)$ metres/second for $0 \le t \le 3$ where t is measured in seconds. Find the distance the insect has travelled during this time using the Left-end approximation with 6 subdivisions.