

# Areas & Integration

## Problem Set 22

- Find the integral  $\int_0^5 (2x - 4) dx$
  - Find the area between  $y = 2x - 4$  and the  $x$ -axis between  $x = 0$  and  $x = 5$ .
- Find the area between the curve  $y = x^2 + 2x$ , the  $x$ -axis and the lines  $x = -1$  and  $x = 1$ .
- Find the area between the curve  $y = x^3 - 4x$  and the  $x$ -axis between  $x = -\frac{1}{2}$  and  $x = 3$ .
- Determine the area enclosed by the curve  $y = x^3$  and the lines  $x = -1$  and  $x = 1$ .
- Consider  $f(x) = \cos 2x$ .
  - Find the area between  $y = f(x)$  and the  $x$ -axis between  $x = 0$  and  $x = \frac{\pi}{4}$ .
  - Find the area between  $y = f(x)$  and the  $x$ -axis between  $x = 0$  and  $x = \frac{\pi}{2}$ .
  - Find the area between  $y = f(x)$  and the  $x$ -axis between  $x = -\frac{\pi}{2}$  and  $x = \frac{\pi}{2}$ .
  - Find the integral  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos 2x dx$
- Find the area enclosed by  $y = x$ ,  $y = \frac{1}{x}$  and the  $x$ -axis for  $0 \leq x \leq 3$ .
- Find the following integrals by considering the areas they represent geometrically.
  - $\int_0^1 x dx$
  - $\int_{-2}^2 \sqrt{4 - x^2} dx$
  - $\int_0^2 \sqrt{4 - x^2} dx$
- Find the area between the two curves  $y = 3 - x^2$  and  $y = x + 1$ .
- Find the area between the parabolas  $y = x^2$  and  $y = x(4 - x)$ .
- Find the area between the two curves  $y = x^3 - 4x$  and  $y = 5x$ .
- Find the area between the two curves  $y = \sin x$  and  $y = \sin 2x$  for  $0 \leq x \leq \pi$ .

12. A car starts from rest at  $s = 4$  metres from the origin, and has velocity at time  $t$  (measured in seconds) given by  $v(t) = t^2 - 5t$ .
- (a) Find the displacement function for the car.
  - (b) Find the displacement at time  $t = 7$  seconds.
13. Find the displacement function of a particle moving with velocity  $v(t) = \cos \pi t$  along a straight line, when  $s(0) = 4$ .
14. A particle is moving along a straight line with acceleration given by the function  $a(t) = 2t - 4$  where  $t$  is the time in seconds. Its initial velocity is  $3m/s$ .
- (a) Find the velocity function at time  $t$ .
  - (b) Find the displacement during the time period  $1 \leq t \leq 10$ .
  - (c) Find the distance travelled during the time period  $1 \leq t \leq 10$
15. A ball is thrown directly upward from a point 8 metres above ground with initial velocity  $49m/s$ . If acceleration due to gravity is  $9.8m/s^2$ , how high will the ball travel?