

# Introduction to Integration

## Problem Set 20

1. Consider the area under the function  $f(x) = x$  between  $x = 0$  and  $x = 1$ .

- (a) Estimate the area by calculating the lower and upper sums using 4 sub divisions (ie:  $L_4$  and  $U_4$ ).
- (b) Using your knowledge of triangles, what is the true area?

2. Find each of the following indefinite integrals.

(a) $\int x^8 dx$	(e) $\int x^{-6} dx$	(i) $\int x^{\frac{1}{6}} dx$
(b) $\int x dx$	(f) $\int \frac{1}{x^2} dx$	(j) $\int x^{-\frac{1}{10}} dx$
(c) $\int x^{20} dx$	(g) $\int \frac{1}{x^7} dx$	(k) $\int x^{-\frac{3}{4}} dx$
(d) $\int x^{-4} dx$	(h) $\int x^{\frac{1}{3}} dx$	(l) $\int \frac{1}{\sqrt{x}} dx$

3. Evaluate each of the following indefinite integrals.

(a) $\int 3x dx$	(d) $\int 5x^{-6} dx$	(g) $\int \frac{5}{\sqrt{x}} dx$
(b) $\int 10x^9 dx$	(e) $\int 3x^{\frac{1}{6}} dx$	(h) $\int \frac{4}{x^2} dx$
(c) $\int -2x^3 dx$	(f) $\int \sqrt{5x} dx$	(i) $\int \frac{1}{4x^2} dx$

4. Find the following integrals.

(a) $\int (x^2 + x) dx$	(d) $\int (1 - 2x - x^3) dx$
(b) $\int (5 - x) dx$	(e) $\int (4x^{-2} + 7) dx$
(c) $\int (x^2 + 3x + 2) dx$	(f) $\int \left( \sqrt{x} - \frac{5}{x^2} \right) dx$

5. Evaluate the following integrals.

(a) $\int (x + 1)(x + 7) dx$	(d) $\int \frac{x + 5}{x^3} dx$
(b) $\int (x - 3)^2 dx$	(e) $\int \frac{1 - x}{\sqrt{x}} dx$
(c) $\int (1 - 4x)^2 dx$	(f) $\int \frac{x - \sqrt{x}}{3x} dx$

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Answers:

$$2a) \frac{x^9}{9} + C, \quad b) \frac{x^2}{2} + C, \quad c) \frac{x^{21}}{21} + C, \quad d) \frac{-x^{-3}}{3} + C, \quad e) \frac{-x^{-5}}{5} + C, \quad f) \frac{-1}{x} + C, \quad g) \frac{-x^{-6}}{6} + C, \quad h) \frac{3}{4}x^{\frac{4}{3}} + C, \\ i) \frac{6}{7}x^{\frac{7}{6}} + C, \quad j) \frac{10}{9}x^{\frac{9}{10}} + C, \quad k) 4x^{\frac{1}{4}} + C, \quad l) 2x^{\frac{1}{2}} + C$$

$$3a) \frac{3x^2}{2} + C, \quad b) x^{10} + C, \quad c) \frac{-x^4}{2} + C, \quad d) -x^{-5} + C, \quad e) \frac{18}{7}x^{\frac{7}{6}} + C, \quad f) \frac{2\sqrt{5}}{3}x^{\frac{3}{2}} + C, \quad g) 10x^{\frac{1}{2}} + C, \\ h) \frac{-4}{x} + C, \quad i) \frac{-1}{4x} + C$$

$$4a) \frac{x^3}{3} + \frac{x^2}{2} + C, \quad b) 5x - \frac{x^2}{2} + C, \quad c) \frac{x^3}{3} + \frac{3x^2}{2} + 2x + C, \quad d) x - x^2 - \frac{x^4}{4} + C, \quad e) \frac{-4}{x} + 7x + C, \\ f) \frac{2}{3}x^{\frac{3}{2}} + \frac{5}{x} + C$$

$$5a) \frac{x^3}{3} + 4x^2 + 7x + C, \quad b) \frac{x^3}{3} - 3x^2 + 9x + C, \quad c) x - \frac{8x^3}{3} + \frac{16x^5}{5} + C, \quad d) \frac{-1}{x} - \frac{5}{2x^2} + C, \quad e) 2x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C, \\ f) \frac{x}{3} - \frac{2}{3}x^{\frac{1}{2}} + C$$