Applications of Differentiation

Problem Set 19

1. Sketch the following curves using calculus and show all important features.

    (a) $y = x^4 - 2x^2 + 7$ (b) $y = -3x^5 - 10x^3 - 18x + 31$

2. A rectangular paddock needs to be constructed using a straight river as one side and 200 metres of fencing material for the other 3 sides. Find the dimensions of the paddock which will maximise the area enclosed.

3. A manufacturer wishes to maximise the volume of a cylindrical can that can be made out of 600π square centimetres of sheet metal. Find the dimensions of the can of greatest volume.

4. A certain rectangular poster requires $96cm^2$ for the printed message and must have a margin around the area of print. The top and bottom margins are required to be $3cm$ and the side margins are required to be $2cm$. Find the overall dimensions of the poster if the amount of paper used is a minimum.

5. A company estimates that the cost (in dollars) of producing x items is $C(x) = 2600 + 2x + 0.001x^2$.

    (a) What is the marginal cost when $x = 1000$.
    (b) What is the average cost when $x = 1000$.
    (c) Find the level of production which minimises the average cost.

6. A firm estimates that the cost (in dollars) of producing x units is $C(x) = 3400 + 4x + 0.002x^2$.

    (a) What is the marginal cost when $x = 500$.
    (b) What is the minimum average cost and at what production level is this achieved?

7. A ball is thrown vertically up in the air and its position relative to the ground after t seconds is given by the functions $h(t) = 100t - 5t^2$.

    (a) Find the velocity and acceleration functions.
    (b) What is the initial velocity?
    (c) For how long will the ball rise?
    (d) How high will the ball rise?
    (e) Describe the motion of the ball at time $t = 5$.
    (f) Find the time for the ball to reach the ground.

8. Consider a particle moving in a straight line with displacement given by $s(t) = \dfrac{t^3}{3} - 3t^2 + 5t + 4$, where s is measured in metres and t in seconds.

    (a) What is the initial velocity of the particle?

    (b) What is the velocity and acceleration at $t = 4$?

    (c) Describe the motion of the particle at $t = 4$.

    (d) When does the particle come to rest?