

Differentiation & Curve Sketching

Problem Set 18

1. Find the second derivative of $f(x) = (x^2 + 1)^4$.

2. Consider the function $f(x) = x^3 - 12x$.

(a) Find the stationary points of $y = f(x)$ and use the second derivative test to classify them.

(b) Find the points of inflexion of $y = f(x)$.

3. Sketch the following curves.

(a) $y = x^4 - 8x^2 + 16$

(b) $y = x^4 - 2x^3 + 2$

4. Differentiate the following functions.

(a) $y = e^x + x^5$

(b) $y = \sin x + \cos x$

(c) $y = 3 \log x - 2e^x + 4 \tan x$

5. Use the product rule to differentiate the following functions.

(a) $f(x) = x^2 e^x$

(b) $g(x) = x \ln x$

(c) $h(x) = e^x \sin x$

6. Use the quotient rule to differentiate the following functions.

(a) $f(x) = \frac{e^x}{1+x}$

(b) $g(x) = \frac{2\sqrt{x}}{\ln x}$

7. Use the chain rule to differentiate the following functions.

(a) $y = e^{3x}$

(f) $y = \sin(2 - 3x)$

(k) $y = \log(\sin x)$

(b) $y = e^{10x}$

(g) $y = \cos(x^2)$

(l) $y = (\log x)^3$

(c) $y = e^{x^2}$

(h) $y = \tan(5x + 4)$

(m) $y = \sqrt{e^x + 2}$

(d) $y = e^{x^3+4x}$

(i) $y = \log(3x + 1)$

(n) $y = 3 \cos \sqrt{x}$

(e) $y = \sin 4x$

(j) $y = \ln(x^2 + 4)$

8. Find the derivative of the following functions.

(a) $y = \cos 3x$

(d) $y = \log(\cos x)$

(b) $y = e^{x^3}$

(e) $y = \sin(5x^2 - 2x)$

(c) $y = \ln(2x^5)$

(f) $y = (\tan x)^2$

9. Differentiate the following functions.

(a) $y = \sin^2 x + \sin(x^2)$

(b) $y = \sin(x \log x)$

(c) $y = \log(xe^{-x})$

10. Find the derivative of the exponential function $y = 2^x$.

11. Find the tangent and normal to the curve $y = \cos^2 x$ at the point $\left(\frac{\pi}{4}, \frac{1}{2}\right)$.

12. Sketch the following curves showing all important features.

(a) $y = x^4 - 4x^3 - 2$

(b) $y = x + \frac{1}{x+3}$

(c) $y = \frac{1}{1-x^2}$

(d) $y = xe^x$

Ans: 8a) $-3 \sin 3x$, b) $5e^{5x}$, c) $3x^2 e^{x^3}$, d) $-\tan x$, e) $\frac{5}{x}$, f) $(10x+2) \cos(5x^2+2x)$, g) $2 \tan x \sec^2 x$.