

# Trigonometry

## Problem Set 15

1. For each of the following trig equations, find the angle  $\theta$ , for  $0 \leq \theta \leq 2\pi$ .

(a)  $\sin \theta = -\frac{1}{2}$

(d)  $\tan \theta = 1$

(b)  $\cos \theta = -\frac{1}{\sqrt{2}}$

(e)  $\cos \theta = \frac{1}{2}$

(c)  $\sin \theta = \frac{\sqrt{3}}{2}$

(f)  $\tan \theta = -\frac{1}{\sqrt{3}}$

2. Find all solutions for each of the following equations in the intervals given, showing every step of your argument.

(a)  $\sin(x) = -\frac{\sqrt{3}}{2}, \quad 0 \leq x \leq 2\pi.$

(b)  $\sin(x) = -\frac{\sqrt{3}}{2}, \quad -\pi \leq x \leq \pi.$

(c)  $\tan(x) = \sqrt{3}, \quad 0 \leq x \leq 4\pi.$

(d)  $\cos(x) = \frac{\sqrt{3}}{2}, \quad -\pi \leq x \leq \pi.$

3. Solve  $\tan^2 \alpha = \frac{1}{3}$ , for  $0 \leq \alpha \leq 2\pi$ .

4. Sketch the following trig functions, and write down their amplitude (if relevant) and period.

(a)  $y = 3 \cos(x)$

(e)  $y = \cos(3x)$

(b)  $y = 1 + \tan(x)$

(f)  $y = 4 \sin 3x$

(c)  $y = 2 - \cos(x)$

(d)  $y = \cos\left(x + \frac{\pi}{2}\right)$

(g)  $y = 4 \sin 3(x - \pi)$

5. As a wave passes by an offshore pylon, the height of the water is modelled by the function  $h(t) = 3 \cos\left(\frac{\pi}{10}t\right)$ , where  $h(t)$  is the height in metres above the mean sea level at time  $t$ .

(a) Find the period of the wave.

(b) Find the wave height (ie. the vertical distance between the trough and the crest of the wave).

6. Use the addition formulas to find the exact value of each expression.

(a)  $\cos(75^\circ)$

(b)  $\cos\left(\frac{7\pi}{12}\right)$

(c)  $\sin\left(\frac{\pi}{12}\right)$

7. Prove the identity  $\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$ .

8. Find the exact value of  $\cos\left(\frac{2\pi}{3}\right)$  using the appropriate double angle formula.

9. Suppose  $\sin x = \frac{5}{13}$  and  $\frac{\pi}{2} \leq x \leq \pi$ .

Find the exact value of  $\cos x$  and hence find the exact value of  $\sin 2x$ .

10. Suppose  $\sin x = \frac{3}{5}$  and  $x$  is in the first quadrant. Find the exact values of  $\sin 2x$ ,  $\cos 2x$  and  $\tan 2x$ .

11. Simplify the following expressions.

(a)  $\frac{\sin x \sec x}{\tan x}$

(b)  $\frac{\sec x - \cos x}{\tan x}$

12. Verify the following identities.

(a)  $\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = 1$

(b)  $\tan 2\theta = \frac{2 \sin \theta \cos \theta}{\cos 2\theta}$

(c)  $\frac{\cos \theta}{1 - \sin \theta} = \sec \theta + \tan \theta$