

Polynomials

Problem Set 11

1. Find the zeros of the following quadratics and sketch their graphs.

(a) $y = (x - 7)(x + 3)$

(b) $y = (x + 3)^2$

(c) $y = -3(9x - 5)(2x + 1)$

(d) $y = 10 + 3x - x^2$.

(e) $y = x^2 - 6x + 9$

2. Sketch the graphs of the following polynomials.

(a) $y = (x + 1)(x - 3)(x + 4)$

(d) $y = x^3 + 3x^2 - x - 3$

(b) $y = -2x(x - 2)(x - 3)(x - 4)$

(e) $y = x^4 - 3x^3 - 4x^2$

(c) $y = (x - 4)(x + 3)^2(x - 1)^3$

(f) $y = x^5 - 4x^4 + 4x^3$

3. Suppose $p(x) = x^3 + 2x^2 + 10$ and $k(x) = 5x^2$.

(a) Find $p(x) + k(x)$

(b) Find $p(x) - k(x)$

(c) Find $p(x)k(x)$

(d) Evaluate $p(1)$ and $k(0)$

4. Let $p(x) = ax^4 - 7x^2 + d$ and $g(x) = 5x^4 + kx^2 - mx - 1$. For $p(x) = g(x)$, find the value of the coefficients a, k, m and d .

5. Find a and b where $a(x - 2) + b(x + 3) = x$

6. Divide the polynomial $3x^3 + 5x^2 - x + 4$ by $x + 2$.

7. Consider $p(x) = x^3 + 4x^2 - 3x - 7$.

(a) Use polynomial long division to divide the polynomial $p(x)$ by $f(x) = x - 1$.

(b) Express $p(x)$ in the form $p(x) = f(x)q(x) + r(x)$ for quotient $q(x)$ and remainder $r(x)$

8. Find the remainder when $p(x) = 3x^3 + 5x^2 - x + 4$ is divided by $x + 2$.

9. Find the remainder when $p(x) = 2x^4 - x^3 + 1$ is divided by $x + 3$.

10. Use the Remainder Theorem to show that $x + 2$ is a factor of $p(x) = x^5 + 2x + 36$.

11. Show that $x - 3$ is a factor of $p(x) = 3x^4 - 8x^3 - 14x^2 + 31x + 6$.

12. Show that $x = 3$ is a root of the polynomial $p(x) = x^3 - x^2 - 41x + 105$ and find all other roots.

13. Guess 1 real root of $p(x) = 2x^3 - 3x^2 - 23x + 12$ and then solve $p(x) = 0$.

14. Consider the polynomial $p(x) = 18x^3 + 3x^2 - 4x - 1$.

(a) Find one root of this polynomial using trial and error.

(b) Hence factorise this polynomial.

(c) Find all solutions of $p(x) = 0$.

(d) Sketch the graph of $p(x)$.