Logs & Exponentials

Problem Set 9

1. Refresh working with exponentials by doing the following questions.

    (a) Simplify $64^{-\frac{1}{6}}$
    (b) Simplify $2^{3x} \times (\sqrt{8})^x$.
    (c) Solve $\left(\dfrac{1}{3}\right)^x \times 27 = 9^x$.

    (d) Simplify $(32x^5)^{\frac{2}{5}} \times \left(\dfrac{25}{x^4}\right)^{\frac{1}{2}}$

2. Refresh working with logarithms by doing the following questions.

    (a) Find the exact value of $\log_5 \left(\dfrac{1}{\sqrt{125}}\right)$.

    (b) Simplify $\left(\log\left(\dfrac{u^3}{v}\right) + \log\left(\dfrac{v}{u}\right)\right) \div \log(\sqrt{u})$.

    (c) Solve $2\ln(x) = \ln(x+6)$.
    (d) Solve $7^x = 12$.
    (e) Solve $e^{9x-1} = 11$

3. Sketch the following functions.

    (a) $f(x) = \log(x) + 2$ (b) $f(x) = \log(x+2)$ (c) $f(x) = \log(x+2) - 1$

4. Show the functions $f(x) = 5e^x$ and $g(x) = \log\left(\dfrac{x}{5}\right)$ are inverses.

5. Find the inverse of $f(x) = e^{x+3}$ and sketch both functions on the same graph.

6. Solve the following equations.

    (a) $e^{3x} = 15$ (b) $12e^{3x} = 15$ (c) $512 - 3e^{-7x} = 210$

7. Suppose $f(x) = 3e^{2x}$.

    (a) Find the value of $f(x)$ when $x = 4$.
    (b) Find the value of x when $f(x) = 51$
    (c) Find the value of x when $f(x) = 0.4$

8. $P(t) = 15600e^{0.09t}$ describes the population of a city t years after 1984.

    (a) What will the population be in 1994?
    (b) How long will it take for the population to reach 100,000?

9. The number of bacteria in a population, given by the formula $N(t) = N_0 e^{0.12t}$, has an initial population of 240000, where t is measured in hours. How long will it take for the population of bacteria to reach 250000?

10. When a certain medical drug is administered to a patient, the number of milligrams remaining in the patient's bloodstream after t hours is modelled by $D(t) = 50e^{-0.2t}$.

    (a) How many milligrams in the initial dose?
    (b) How may milligrams will remain in the bloodstream after 3 hours?
    (c) The patient needs to take a second dose of the drug once there is less than 5mg in their bloodstream. How many hours later does the second dose need to be administered?

11. The number of a certain species of frog is modelled by the function $N(t) = 85e^{0.18t}$ where t is measured in years.

    (a) What is the initial population of frogs?
    (b) What will the population be after 3 years?
    (c) After how many years will the number of frogs reach 600?

12. A hot bowl of soup is served at a dinner party. It starts to cool according to Newton's Law of Cooling so that its temperature at time t is given by $T(t) = 18 + 62e^{-0.05t}$ where t is measured in minutes and T is measured in degrees Celsius(C).

    (a) What is the initial temperature of the soup?
    (b) What is the temperature after 10 minutes?
    (c) After how long will the temperature be 37 degrees C?
    (d) Make a graph of the temperature as a function of time.

13. Phosphorous has a half life of 14 days. Suppose a sample of this substance has a mass of 300 mg.

    (a) Find the initial value and growth constant for this exponential decay function.
    (b) Hence write down the exponential decay function which models the amount of Phosphorous that remains in the sample after t days.

14. The half life of cesium-137 is 30 years. Suppose we have a 100 g sample.

    (a) Find a function that models the mass remaining after t years.
    (b) How much of the sample will remain after 4000 years?
    (c) After how long will only 18 g of the sample remain?

15. The half life of strontium-90 is 28 years. How long will it take a 50 mg sample to decay to a mass of 32 mg?

16. A wooden artifact from an ancient tomb contains 65% of the carbon-14 that is present in living trees. The half-life of carbon-14 is 5730 years. How long ago was the artifact made?