8) Solve $\longleftarrow$ we are given an equation

Aim $x =$

a) $\log_5 x = \log 3 - \log_5 2$

ie: $\log_5 x = \log_5 \left(\frac{3}{2}\right)$

$\therefore x = \frac{3}{2}$

we are NOT cancelling logs

Instead we are saying

$\log A = \log B \Rightarrow A = B$

b) $2\log_7 3 + \log_7 4 - \log_7 6 = \log_7 x$

ie: $\log_7 3^2 + \log_7 4 - \log_7 6 = \log_7 x$

ie: $\log_7 \left(\frac{9 \times 4}{6}\right) = \log_7 x$

ie: $\log_7 6 = \log_7 x$

$\therefore x = 6$

c) $\log_b (x+1) + \log_b 3 = \log_b 10$

$\log_b 3(x+1) = \log_b 10$

$\therefore 3(x+1) = 10$

$3x + 3 = 10$

$3x = 7$

$x = 7/3$

d) $2\log_b x = \log_b 4$

ie: $\log_b x^2 = \log_b 4$

$\therefore x^2 = 4$

$x = \pm 2$ ← But x cant be negative

$\therefore x = 2$

Since log is not defined for some values we should check our answers.

e) $2\log_b x = \log_b (3x+4)$

$\therefore \log_b x^2 = \log_b (3x+4)$

$\therefore x^2 = 3x+4$

ie: $x^2 - 3x - 4 = 0$

$(x+1)(x-4) = 0$

$\therefore x = -1, 4$

Ignore neg $\therefore x = 4$

f) $\log_2 (x+1) - \log_2 x = 3$

ie: $\log_2 \left(\frac{x+1}{x}\right) = 3$

$\therefore \frac{x+1}{x} = 2^3$ ← change to exp form

$\frac{x+1}{x} = 8$

$x+1 = 8x$

$1 = 7x$

$x = 1/7$

Natural Log

- Logs can have many bases

$\log_2 x$, $\log_5 x$, ... etc.

$\log_{10} x$ ← common (used in economics)

$\log_e x$ ← most common in Maths

Remember $e = 2.718\ldots$ (irrational number) that gives us nice results in Calculus.

- In maths we like to work in $\log_e x$.
- called the Natural Log.
- We say $\log_e x = \ln x = \log x$

note: on calculator $\boxed{\log} = \log_{10}$

$\boxed{\ln} = \log_e$

- We work with natural log in the same way.

eg 9 a) $\log_e e^2 = 2$

b) $\ln x^2 - \ln xy = \ln\left(\frac{x^2}{xy}\right)$

$= \ln\left(\frac{x}{y}\right)$

eg 10) Solve a) $2\ln x = \ln(3x+10)$

ie: $\ln x^2 = \ln(3x+10)$

ie: $x^2 = 3x+10$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5, -2$

Ignore neg

$\therefore x = 5$

b) Solve $\ln(x+3) = 4$

$\therefore x+3 = e^4$

$x = e^4 - 3$

c) Solve $e^{x+1} = 2$

$\therefore$ $x+1 = \ln 2$

$x = \ln 2 - 1$

d) Solve $5e^{x} = 12$

$e^{x} = \frac{12}{5}$

$\therefore$

$x = \ln\left(\frac{12}{5}\right)$

e) Solve $5^{x} = 23$

method 1 : $x = \log_5 23 \longleftarrow$ Fine But we prefer Base e.

method 2 : $5^{x} = 23$

$\log 5^{x} = \log 23$

$x \log 5 = \log 23$

$x = \frac{\log 23}{\log 5}$

f) Solve $7^{x} = 15$

$\log 7^{x} = \log 15$

$x \log 7 = \log 15$

$x = \frac{\log 15}{\log 7}$

The function "Log"

- we have been talking about Logs as the thing that undoes exponentials.

  ie: The function $y = \log_b x$ undoes exp $y = b^x$

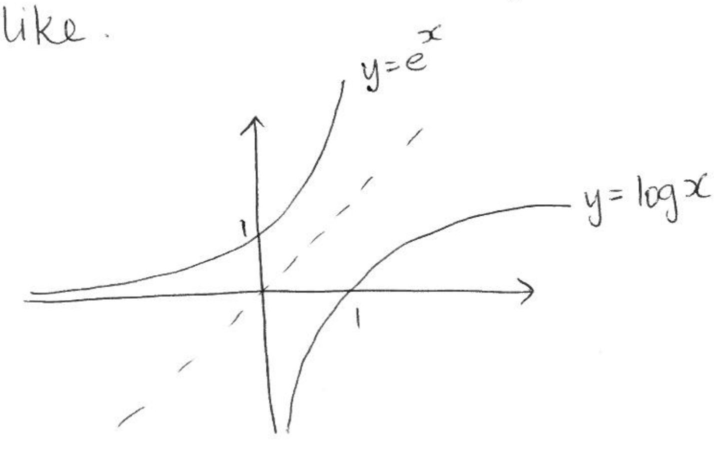
  Recall - This is what an inverse does.

  So Log is the inverse of exponentials.

  This is the actual definition of log

- Knowing about the property of inverse functions means we also know what the graph of log should look like.

ie:



We can see the properties of log from its graph.

Dom = $(0, \infty)$ (can't take 0 or neg values)

Range = $\mathbb{R}$

$\log 1 = 0$

$\log x$ is increasing but increasing slowly