Refresher

$$\boxed{x = b^y \iff y = \log_b x}$$

$$2^3 = 8 \iff \log_2 8 = 3$$

- log gives us a power
- log undoes an exponential
- $\log_2 8$ ← "log base 2 of 8"

  applying the rule of undoing powers

Properties :

$$\log_b A + \log_b B = \log_b(AB)$$

$$\log_b A - \log_b B = \log_b\left(\frac{A}{B}\right)$$

$$\log_b A^n = n \log_b A$$

eg 1) $\log_2 x + \log_2 3 - \log_2 5 = \log_2\left(\frac{3x}{5}\right)$

2) $\log_3 4 + \log_3 x^2 = \log_3 (4x^2)$

$$= \log_3 (2x)^2$$

$$= 2\log_3(2x)$$

Other properties

- Two other properties come about because of the special relationship between logs and exponentials ie: They undo each other.

$$\therefore \quad \boxed{\log_b b^x = x}$$

← putting exp into log
← base of exp is same as base of log so they undo.

eg: a) $\log_2 2^3 = 3$

b) $\log_2 2 = 1$

c) $\log_3 81 = \log_3 3^4 = 4$

d) $\log_5 25 = \log_5 5^2 = 2$

$$\boxed{b^{\log_b x} = x}$$

← putting log into exp when bases are same they undo.

eg a) $2^{\log_2 3} = 3$

b) $5^{2\log_5 x}$

$= 5^{\log_5 x^2}$

$= x^2$

Summary of Log Properties:

- $\log_b x = y \iff x = b^y$

- $\log_b A + \log_b B = \log_b (AB)$
- $\log_b A - \log_b B = \log_b \left(\frac{A}{B}\right)$
- $\log_b A^n = n \log_b A$
- $\log_b b^x = x$
- $b^{\log_b x} = x$

Because of the relationship between logs + exp we also know

- $\log_b 1 = 0$ (since any exp $b^0 = 1$)
- $\log_b 0 =$ undefined (since an exp is never 0)
- $\log_b (\text{Neg}) =$ undefined (since an exp is never neg).

Examples

6) Evaluate

a) $\log_7 49 = \log_7 7^2 = 2$

b) $\log_3 \left(\frac{1}{27}\right) = \log_3 27^{-1}$

$= \log_3 3^{-3}$

$= -3$

c) $\log_5 \sqrt{125} = \log_5 (125)^{1/2}$

$= \log_5 (5^3)^{1/2}$

$= \log_5 5^{3/2}$

$= 3/2$

d) $\sqrt{\log_5 125} = \sqrt{\log_5 5^3}$

$= \sqrt{3}$

e) $\log_5 (-25)$ = undefined (cant take log of neg)

f) $\log_{10} (0.001) = \log_{10} 10^{-4}$

$= -4$

7) Simplify ← ie: usually Try + write as single log

a) $\log_2 x + 4\log_2(3x) = \log_2 x + \log_2 (3x)^4$

$= \log_2 x + \log_2(81x^4)$

$= \log_2(81x^5)$

b) $\frac{1}{3}\log_a 5 - 3\log_a x - 4\log_a y$

$= \underbrace{\log_a 5^{1/3} - \log_a x^3} - \log_a y^4$

$= \log_a\left(\frac{5^{1/3}}{x^3}\right) - \log_a y^4$

$= \log_a\left(\frac{5^{1/3}}{x^3 y^4}\right)$

c) $\log_4 \frac{x^4}{y^2} - \log_4 \frac{y^2}{x} = \log_4\left(\frac{x^4}{y^2} \div \frac{y^2}{x}\right)$

$= \log_4\left(\frac{x^4}{y^2} \times \frac{x}{y^2}\right)$

$= \log_4\left(\frac{x^5}{y^4}\right)$

d) $\log_3 (x+1) + \log_3 y - \log_3 xy$

$= \log_3 \frac{(x+1)y}{xy}$

$= \log_3 \left(\frac{x+1}{x}\right)$

Note: • Don't cancel x's (remember cancel factors NOT terms)

$\cancel{\log_3 \left(\frac{\cancel{x}+1}{\cancel{x}}\right)}$

• $\log_3 (x+1)$ ← Remember this is log OF $(x+1)$

∴ Dont expand.

$\cancel{\log_3 (x+1) = \log_3 x} + \log_3 1$

e) $\underbrace{\log_b (x^2-y^2) - \log_b (x-y)} - \log_b (x+y)$

$= \log_b \frac{x^2-y^2}{x-y} - \log_b (x+y)$

$= \log_b \left(\frac{x^2-y^2}{(x-y)(x+y)}\right)$

$= \log_b \left(\frac{x^2-y^2}{x^2-y^2}\right)$

$= \log_b 1$

$= 0$