Logarithms

We know $2^3 = ?$

What if we are asked $2^? = 8$

↑

The problem of finding a
is finding a logarithm

Here we know $? = 3$ + we say $\log_2 8 = 3$

↑

"log base 2 of 8"

So $\log_2 8$ is saying "what power of 2 gives 8"

$\therefore \log_2 8 = 3$

$\log_5 25$ ← "what power of 5 gives 25" (ie: $5^? = 25$)

$\therefore \log_5 25 = 2$

$\log_{10} 1000$ ← "what power of 10 gives 1000" (ie: $10^? = 1000$)

$\therefore \log_{10} 1000 = 3$

So $2^3 = 8 \iff \log_2 8 = 3$

Rule : $\boxed{x = b^y \iff y = \log_b x}$

exponential form

logarithmic form

we say "log base b of x"

Notice : · Base of exp is the base of log

· Power of exp is the answer of log.

$\log_b x$ ← this is log OF x ← ie: we are applying a rule to x.

(not multiplying)

The rule is the undoing of powers

So $\log_b x$ is a function

Logs and exponentials have a special relationship

→ they undo each other

→ ie: they are inverses

eg1) a) We know $3^2 = 9$ $\leftarrow$ exp form

So $2 = \log_3 9$ $\leftarrow$ log form

(using formula $y=2, b=3, x=9$)

b) $4^3 = 64 \iff \log_4 64 = 3$

c) $5^2 = 25 \iff \log_5 25 = 2$

eg2) Find x

a) $\log_2 x = 4$

$x = 2^4 = 16$

b) $\log_9 x = 1/2$

$x = 9^{1/2} = 3$

c) $\log_4 \left(\frac{x}{3}\right) = \frac{1}{2}$

$\frac{x}{3} = 4^{1/2}$

$\frac{x}{3} = 2$

$x = 6$

d) $\log_x 25 = 2$

$\therefore x^2 = 25$

$\therefore x = 5$ (Negative bases dont make sense).

e) $\log_x 64 = 3$

$\therefore x^3 = 64$

$\therefore x = 4$

In all these ques we have been using the relationships between exp + logs.

$$x = b^y \Leftrightarrow y = \log_b x$$

- Remember log gives a power

- $\log_b x$ is $\log_b$ OF x

Not a multiplication.

It is a ~~reile~~ applied to x (just as $\sqrt{\ }$ is a rule)

(Try Prob Set Q1 and Q2)

Properties of Logs

Since Logs represent powers, their properties are the properties of powers.

eg: We know $10^2 \times 10^3 = 10^5$

Powers $2 + 3 = 5$

$\log_{10} 100$ $\log_{10} 1000$ $\log_{10} 100000$

ie: $\log_{10} 100 + \log_{10} 1000 = \log_{10} 100000$

ie: $\boxed{\log_b A + \log_b B = \log_b AB}$

This is the first property

ie: Adding 2 logs with same bases
→ two things get multiplied.

eg a) $\log_5 3 + \log_5 4 = \log_5 (12)$

b) $\log_5 3 + \log_6 4$ ← can't combine since bases are different

c) $\log_5 3 + \log_2 3 + \log_2 4 + \log_5 4$

$= \log_5 3 + \log_5 4 + \log_2 3 + \log_2 4$

$= \log_5 (12) + \log_2 (12)$ ← can't go further:
- bases different
- log "OF" so cant factorise

Try Prob Q3

Rule 2 : $\boxed{\log_b A - \log_b B = \log_b \left(\frac{A}{B}\right)}$

← we can see this in a similary way to addition rule.

eg a) $\log_3 8 - \log_3 2 = \log_3 \left(\frac{8}{2}\right)$

b) $\log_2 9 + \log_2 5 - \log_2 3 = \log_2 \left(\frac{9 \times 5}{3}\right)$

$= \log_2 \left(\frac{45}{3}\right) = \log_2 (15)$

c) $\log_5 (4x^2) - \log_5 (2x) = \log_5 \left(\frac{4x^2}{2x}\right) = \log_5 2x$

(Try Probs Q4)

Rule 3 : $\boxed{\log_b A^n = n \log_b A}$

← can think of this as repeated addition

ie: $n \log_b A = \log_b A + \cdots + \log_b A$

$= \log_b (A \times \cdots \times A)$

$= \log_b A^n$

a) $\log_2 x^3 = 3 \log_2 x$

b) $\log_5 (9x^2) = \log_5 (3x)^2 = 2 \log_5 3x$

This is different to $(\log_5 3x)^2$

(Try Prob Q5)

* There is no rule for multiplying and dividing two logs.