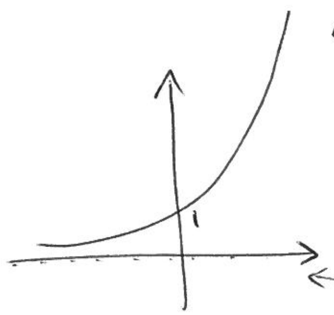
So Far :

Exponentials

eg : $y = 3^x$



Dom = $\mathbb{R}$
Range = $(0, \infty)$

In general $y = ab^x$

eg: Find eqn of exp going through $(-3, \frac{1}{2})$ and $(2, 5)$.

Let $y = ab^x$

$(-3, \frac{1}{2})$: $\frac{1}{2} = ab^{-3}$ — ①

$(2, 5)$: $5 = ab^2$ — ②

From ② : $a = \frac{5}{b^2}$

sub in ① : $\frac{1}{2} = \left(\frac{5}{b^2}\right) b^{-3}$

ie: $\frac{1}{2} = \frac{5}{b^5}$

$\therefore b^5 = 10$

$b = 10^{1/5}$

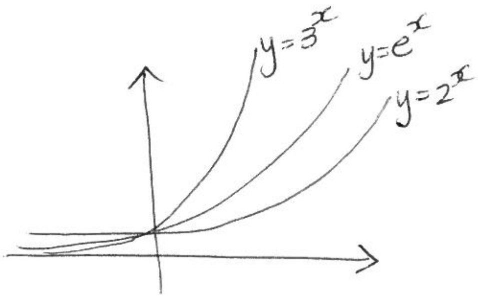
$\therefore$ sub back in : $a = \frac{5}{(10^{1/5})^2} = \frac{5}{10^{2/5}}$

$\therefore y = \left(\frac{5}{10^{2/5}}\right)(10^{1/5})^x = \frac{5}{10^{2/5}} \cdot 10^{x/5} = (5)10^{\frac{x}{5} - \frac{2}{5}}$

The Natural Exponential

- The base on an exponential can be any number but the most common one in maths is Base e.

$e = 2.718...$



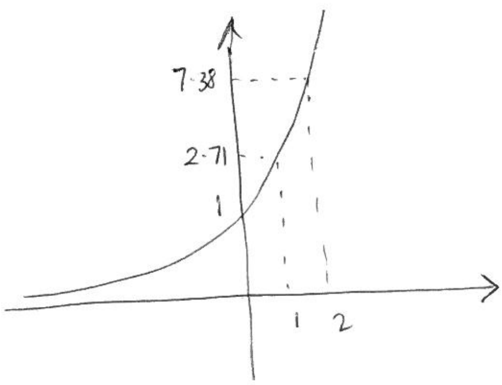
- e is an irrational number (just like π)
- we use it in maths because it gives us nice results in calculus.
- it's defined as the value that $(1+\frac{1}{n})^n$ approaches as n becomes bigger (ie: $n \to \infty$)

$f(x) = e^x$ is called the natural exponential function

- We treat it like any other exponential.

eg:

x	$f(x) = e^x$
-2	$e^{-2} = 0.136$
-1	$e^{-1} = 0.368$
0	1
1	$e^{1} = 2.71$
2	$e^{2} = 7.389$

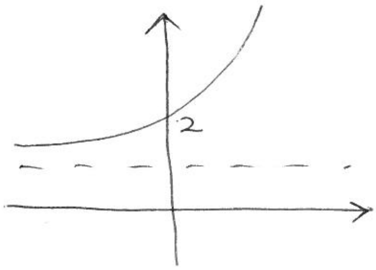


use e^x button on calculator

Modifying Exponentials

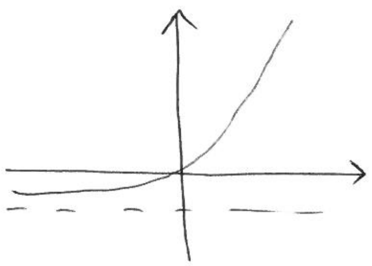
- we can modify exponentials the same way we have modified other graphs.

eg: $f(x) = e^{x} + 1$



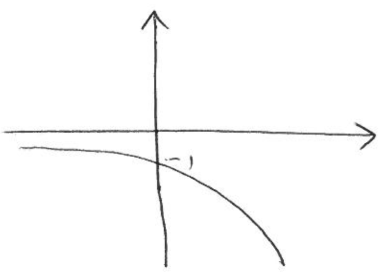
vertical shift up

$f(x) = e^{x} - 1$



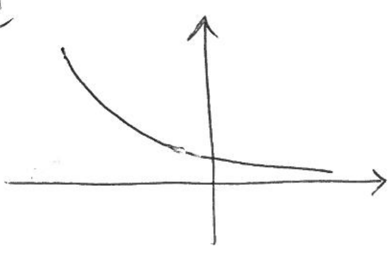
vertical shift down

$f(x) = -e^{x}$



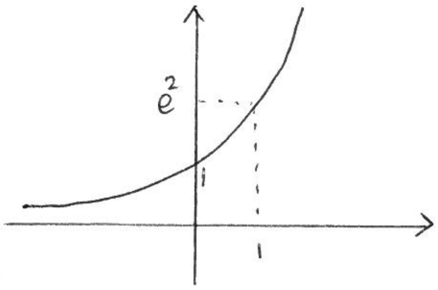
positives now become negative

$f(x) = e^{-x}$



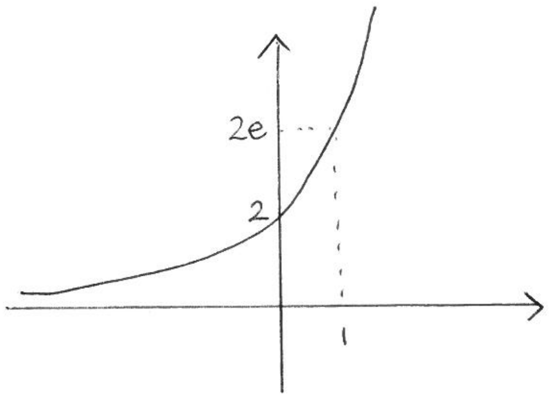
reflection about y-axis.

$f(x) = e^{2x}$



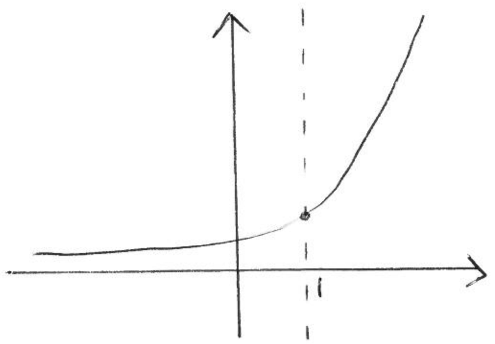
steeper

$f(x) = 2e^{x}$



- y values doubled
- note : when $x=0, y=2$

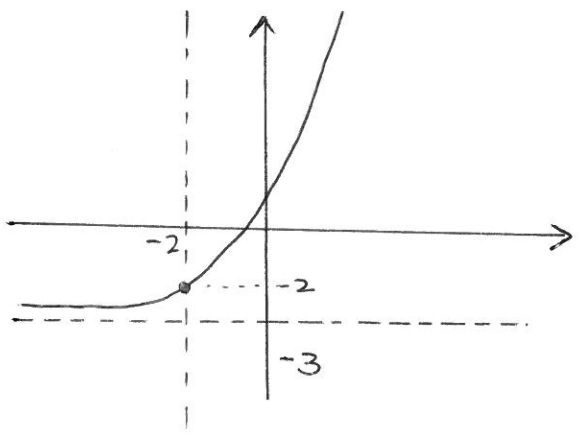
$f(x) = e^{x-1}$



what used to happen at 0
now happens at 1

ie: horiz shift to right

$f(x) = e^{x+2} - 3$



Dom $= \mathbb{R}$
Range $= (-3, \infty)$

The Exponential Equation

- We have seen that exponentials have the form $y = ab^x$.
- A more common form is to use base e

  and write $y = Ae^{kx}$ $A, k =$ constants.

eg: $y = 5e^{2x}$

$y = 3e^{-x}$ ← These are all exponentials

$y = \frac{1}{2}e^{5x}$

$y = 10e^{0.2x}$

Notice we can always put this into the original form

$$\begin{aligned} y &= 10e^{0.2x} \\ &= 10\left(e^{0.2}\right)^x \\ &= \underset{a}{10}\,(\underset{b}{1.2214\ldots})^x \end{aligned}$$

From now on we will mainly work with $y = Ae^{kx}$.

$A, k =$ constants.

$k =$ growth constant - determines how fast/slow model is growing / decaying.

If $k > 0$ eg: $y = Ae^{2x}$  growing

$k < 0$ $y = Ae^{-2x}$  decaying

Notice $A =$ initial value (when $x = 0$, $y = A$)

Eg: $f(x) = 7e^{-3x}$

(a) Evaluate $f(\frac{1}{2})$

(b) What is the y-intercept of this function

(c) Sketch the graph of this function.

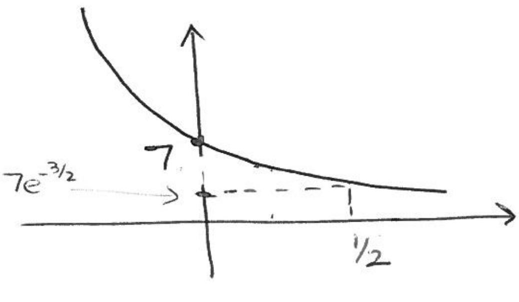
(d) What happens as x gets larger.

(a) $f(\frac{1}{2}) = 7e^{-3(\frac{1}{2})} = 7e^{-3/2}$ $(\simeq 1.562)$

(b) y intercept $\rightarrow$ Let $x = 0$ $\therefore$ $y = 7e^{0} = 7$

c) Sketch $y = 7e^{-3x}$

exp decay



d) As ~~y~~ x gets larger $y = 7e^{-3x} = \frac{7}{e^{3x}}$

$x \rightarrow \infty$, $e^{3x} \rightarrow \infty$ so $\frac{1}{e^{3x}} \rightarrow 0$.

$\therefore$ $y = \frac{7}{e^{3x}}$ gets smaller

$\therefore$ $f(x) \rightarrow 0$.

Consider populations

(eg: population of bacteria)

- populations tend to grow exponentially
- typical population model looks like

$$N(t) = N_0 e^{kt}$$

N = number in population

t = time

N_0, k = constants.

N_0 = initial population

ie: when $t=0$, $N = N_0$

eg 9) Here $N(t) = N_0 e^{0.12t}$

- This is a growth model since $k = 0.12$ is pos.
- As time progresses, population will grow exponentially according to this equation.
- We are given info about the initial value

9. The number of bacteria in a population, given by the formula $N(t) = N_0 e^{0.12t}$, has an initial population of 240000. How many bacteria will be present after 5 hours?

Initial pop is 240000 → ie: when $t = 0$, $N = 240000$

ie: $N = N_0 e^{0.12t}$ → $240\,000 = N_0 e^{0.12(0)}$

$\therefore$ $N_0 = 240\,000$

So our model looks like $N(t) = 240000\, e^{0.12t}$

We want N after 5 hours

→ ie: want N when $t=5$

$\therefore$ $N(5) = 240\,000\, e^{0.12(5)}$

$= 437\,308$

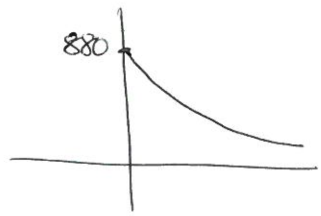
10. The demand D for a specific product in items per month is given by $D(x) = 880e^{-0.18x}$, where x is the price in dollars of the product.

(a) What will be the monthly demand when the price is \$10?

(b) What will be the monthly demand when the price is \$18?

(c) What will happen to the demand as the price increases without bound?

$D(x) = 880\,e^{-0.18x}$



- Firstly notice this is decaying
- Initially $D = 880$ (ie: when $x=0$)
- So as price increases, demand gets less.

a) want D when $x=10$: $D = 880\,e^{-0.18(10)} = 145$ (nearest whole n°)

b) want D when $x=18$: $D = 880\,e^{-0.18(18)} = 34$

c) We can see as x gets bigger, $e^{-0.18x}$ gets smaller

ie! as $x \to \infty$, $e^{-0.18x} = \dfrac{1}{e^{0.18x}} \to 0$

$\therefore$ Demand approaches 0