# Exponentials

## Refresh Indices

Recall : $3 \times 3 \times 3 \times 3 = 3^{4}$ ← index = power

↑ base

In general $a^n = \underbrace{a \times a \times \cdots \times a}_{n \text{ times}}$.

Power Rules :

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^m} = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

$$(ab)^m = a^m b^m$$

* No rule for +/-
* Rules only work when bases are the same.

Also

$$a^0 = 1$$

$$a^1 = a$$

$$a^{-1} = \frac{1}{a}$$

$$a^{1/2} = \sqrt{a}$$

$$a^{1/3} = \sqrt[3]{a}$$

← fractional powers = roots

eg : $3^2 \times 3^3 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$ ← example of the first rule.

- we need to be comfortable working with powers.

eg a) $2^2 \times 2^3 = 2^5$

b) $2^2 + 2^3$ ← No rule for addition

∴ work out long way $2^2 + 2^3$

$= 4 + 8$

$= 12$

c) $2^2 \times 3^3$ ← different bases ∴ can't use rules

∴ Work out long way $2^2 \times 3^3$

$= 4 \times 27$

$= 108$

d) $2^2 \times 8^3 = 2^2 \times (2^3)^3$

$= 2^2 \times 2^9$

$= 2^{11}$

e) $2^2 \times \left(\frac{1}{2}\right)^3 = 2^2 \times (2^{-1})^3$

$= 2^2 \times 2^{-3}$

$= 2^{-1}$

f) $(2x)^3 = 2^3 x^3 = 8x^3$

g) $(8^{1/3})^3 = 8$

Note: $(8^{1/3})^3 = 8$ ← The third power rule $(a^m)^n = a^{mn}$ is the reason why fractional powers are defined as roots.

↑ Here we need $8^{1/3}$ to be cube root of 8.

ie: $8^{1/3}$ = number that cubes to give 8

$\therefore 8^{1/3} = 2$.

So $(-8)^{1/3}$ = number that cubes to give -8

$= -2$.

But $(-8)^{1/2}$ = number that squares to give -8

Not possible!

$\therefore$ This is undefined

eg h) $8^{2/3} = (8^{1/3})^2$

$= 2^2$

$= 4$

i) $16^{3/2} = (16^{1/2})^3$

$= 4^3$

$= 64$

Evaluate:

Eg 1) a) $\dfrac{5^2\, 8^{1/3}}{5^{-4}\, 2^7} = \dfrac{5^2 \cdot 2}{5^4 \cdot 2^7}$

$= 5^{2-(-4)} \cdot 2^{1-7}$

$= 5^6 2^{-6}$

$= \dfrac{5^6}{2^6}$

$= \left(\dfrac{5}{2}\right)^6$

b) $\dfrac{3^2 \cdot 4^{-2} \cdot 5}{2^{-4} \cdot 3^3 \cdot 25} = \dfrac{3^2 \cdot (2^2)^{-2} \cdot 5}{2^{-4} \cdot 3^3 \cdot 5^2}$

$= \dfrac{3^2 \cdot 2^{-4} \cdot 5}{2^{-4} \cdot 3^3 \cdot 5^2}$

$= 3^{2-3} \cdot 5^{1-2}$

$= 3^{-1} \cdot 5^{-1}$

$= \dfrac{1}{3 \cdot 5}$

$= \dfrac{1}{15}$

Simplify

2) a) $\dfrac{(3x^2)(4x^5)}{x^{1/2}} = 12x^{2+5-1/2}$

$= 12x^{13/2}$

b) $(4x^{1/2}y^{-1})^{7/2} = 4^{7/2}(x^{1/2})^{7/2}(y^{-1})^{7/2}$

$= (4^{1/2})^7 x^{7/4} \cdot y^{-7/2}$

$= \dfrac{2^7 x^{7/4}}{y^{7/2}}$

c) $(5x^2)^3 \left(\dfrac{x^4}{25}\right)^{1/2} = 5^3 \cdot x^6 \cdot \dfrac{(x^4)^{1/2}}{25^{1/2}}$

$= \dfrac{5^3 \cdot x^6 \cdot x^2}{5}$

$= 5^2 \cdot x^8$

d) $\left(\dfrac{2x^{3/4}}{y^{1/3}}\right)^3 \div \left(\dfrac{16}{y^4}\right)^{1/2}$

$= \dfrac{2^3 x^{9/4}}{y} \div \dfrac{16^{1/2}}{y^2}$

$= \dfrac{2^3 x^{9/4}}{y} \times \dfrac{y^2}{4}$

$= \dfrac{2^3}{4} \cdot x^{9/4} \cdot y^{2-1}$

$= 2x^{9/4}y$

We've been looking at expressions that have numbers in the powers. We can also have $x$'s in the powers.

Eg 3) Simplify

a) $2^{3x} \times \dfrac{2^{x-1}}{3^x} = \dfrac{2^{3x+(x-1)}}{3^x}$

$= \dfrac{2^{4x-1}}{3^x}$ ← can't go further since bases are different.

b) $16^x \times 2^{5x} = (2^4)^x \times 2^{5x}$

$= 2^{9x}$

c) $9^{x/2} \times 27^{x/3} = (3^2)^{x/2} \times (3^3)^{x/3}$

$= 3^x \times 3^x$

$= 3^{2x}$

d) $32^{x+2} \div 8^{2x-1} = (2^5)^{x+2} \div (2^3)^{2x-1}$

$= 2^{5x+10} \div 2^{6x-3}$

$= 2^{5x+10-(6x-3)}$

$= 2^{-x+13}$

Solve ← remember this means "find $x$"

eg4) a) $9^x = 3^{x-1}$

ie: $3^{2x} = 3^{x-1}$

ie: $2x = x - 1$

$x = -1$

b) $8^x \times 2^x = 4^{4-2x}$

ie: $(2^3)^x \times 2^x = (2^2)^{4-2x}$

$2^{3x} \times 2^x = 2^{8-4x}$

$2^{4x} = 2^{8-4x}$

$\therefore 4x = 8 - 4x$

$8x = 8$

$x = 1$

c) $25 \times 5^x = \left(\frac{1}{5}\right)^x \times \frac{1}{125}$

ie: $5^2 \times 5^x = 5^{-x} \times 5^{-3}$

$\therefore 5^{2+x} = 5^{-x-3}$

$\therefore 2 + x = -x - 3$

$2x = -5$

$x = -5/2$