Increasing + Decreasing Functions

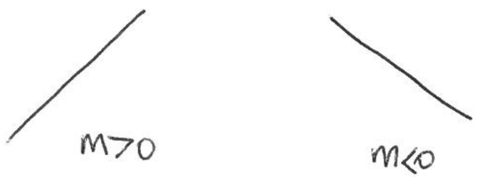
Increasing = going up

as x gets bigger y gets bigger

Decreasing = going down

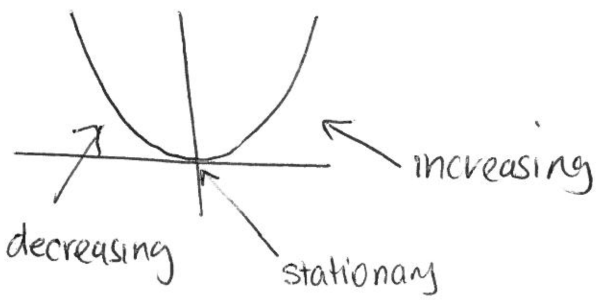
as x gets bigger y gets smalle

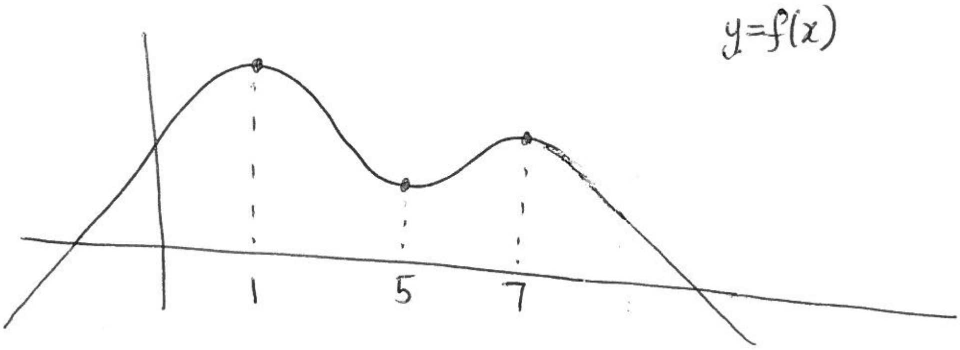
- Lines



are either increasing ($m>0$) or decreasing ($m<0$) (not both)

- $y=x^2$





when $x<1$ $f(x)$ is increasing

$1<x<5$ " " decreasing

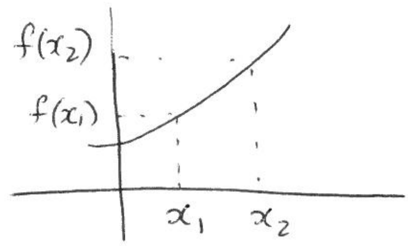
$5<x<7$ " " increasing

$x>7$ " " decreasing.

ie: f increasing on $(-\infty,1)$ and $(5,7)$, f dec on $(1,5)$ and $(7,\infty)$

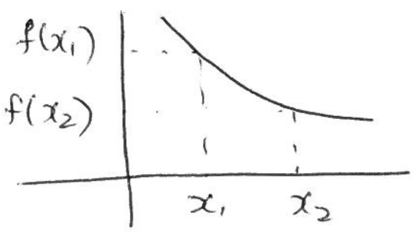
Formally :

$f(x)$ is increasing on interval I if $f(x_1) \leq f(x_2)$ for $x_1 < x_2$ in I

ie: 

(and strictly inc if $f(x_1) < f(x_2)$)

$f(x)$ is decreasing on interval I if $f(x_1) \geq f(x_2)$ for $x_1 < x_2$ in I

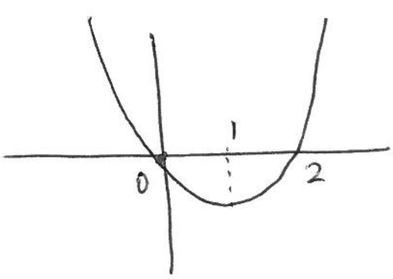


(and strictly dec if $f(x_1) > f(x_2)$).

$f(x)$ is monotonic if its either increasing or decreasing

(But not both).

eg: $g(x) = x^2 - 2x$

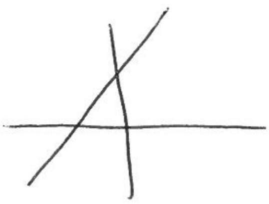


$f(x)$ is strictly dec on $(-\infty, 1)$

$f(x)$ is strictly inc on $(1, \infty)$

It is not monotonic

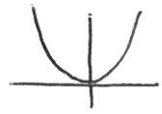
$g(x) = 2x + 1$



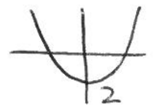
- $f(x)$ is increasing on $\mathbb{R}$
- $f(x)$ is monotonic

Eg 3) $f(x) = |x^2 - 2|$

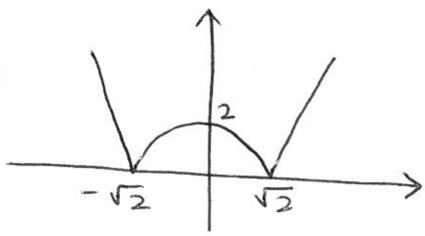
a) Basic fn is parabola $y = x^2$



$\therefore \quad y = x^2 - 2$



So $f(x) = |x^2 - 2|$



x intercepts $\rightarrow$ Let $y = 0$: $|x^2 - 2| = 0$

$$\therefore \quad x^2 - 2 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

y intercepts $\rightarrow$ Let $x = 0$: $y = |0^2 - 2|$

$$= |-2|$$
$$= 2$$

b) From our graph we can see

Domain $= \mathbb{R}$

Range $= \{ y \in \mathbb{R} : y \geq 0 \}$

c) There is symmetry about the y-axis.

$\therefore$ This function is even

We can prove this by using the definition of an even function: $f(x) = f(-x)$.

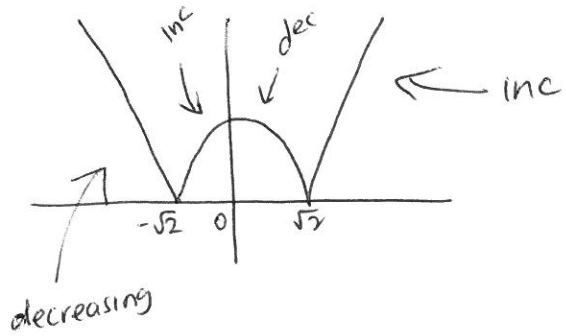
$f(x) = |x^2 - 2|$ and $f(-x) = |(-x)^2 - 2|$

$= |x^2 - 2|$

$= f(x)$.

$\therefore f(x)$ is even.

d)



From graph $f(x)$ is decreasing on $(-\infty, -\sqrt{2})$ and $(0, \sqrt{2})$

$f(x)$ is increasing on $(-\sqrt{2}, 0)$ and $(\sqrt{2}, \infty)$

Composition

= way of combining functions

* Don't forget input-output concept of function

eg: $f(x) = x + 4$

input ↑ ↑ output

"adds 4 to input"

$\therefore$ $f(6) = 6 + 4 = 10$

$f(a) = a + 4$

$f(a^2) = a^2 + 4$

$f(x^2) = x^2 + 4$

$[f(x)]^2 = (x+4)^2$

Don't confuse these two

$f(x+1) = (x+1) + 4$

$f(x+h) = (x+h) + 4$

$f(x^3) = x^3 + 4$

Think of this as substituting another function into f.

$\therefore$ Let $g(x) = x^3$

So $f(g(x)) = f(x^3) = x^3 + 4$

Substituted a function into a function

ie: we have the composition of functions

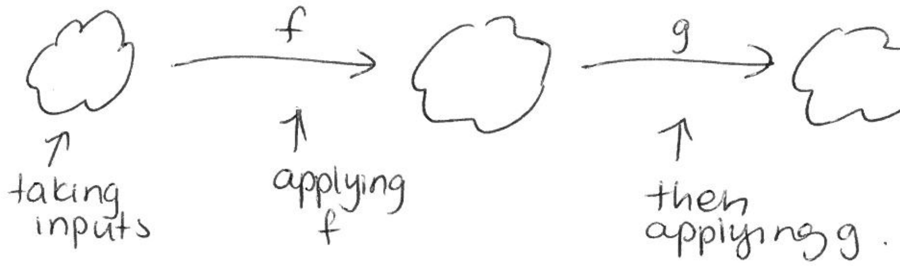
Take another function $h(x) = \frac{1}{x}$ and $f(x) = x+4$, $g(x) = x^3$

so $f(h(x)) = f\left(\frac{1}{x}\right) = \frac{1}{x} + 4$

$$g(h(x)) = g\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 = \frac{1}{x^3}$$

$$g(f(x)) = g(x+4) = (x+4)^3$$

What are we doing : $g(f(x))$



eg : $g(f(2)) = g(6)$

$= 6^3$

$= 216$

← Evaluate from inside out

$g(f(-8)) = g(-4)$

$= (-4)^3$

$= -64$

We say $g(f(x)) = (g \circ f)(x)$

$f(g(x)) = (f \circ g)(x)$

Eg 4) $f(x) = 2x - 1$ $\quad g(x) = x^2 + x$ $\quad h(x) = \frac{1}{x}$

a) $(g \circ f)(3) = g(f(3)) = g(5) = 5^2 + 5 = 30$

b) $(f \circ g)(3) = f(g(3)) = f(12) = 2(12) - 1 = 23$

c) $(g \circ f)(x) = g(f(x))$
$= g(2x-1)$
$= (2x-1)^2 + (2x-1)$

d) $(f \circ g)(x) = f(g(x))$
$= f(x^2+x)$
$= 2(x^2+x) - 1$

Notice
$g \circ f(x) \neq f \circ g(x)$

e) $(h \circ g)(x) = h(g(x))$
$= h(x^2+x)$
$= \frac{1}{x^2+x}$

f) $(g \circ h)(x) = g(h(x))$
$= g\left(\frac{1}{x}\right)$
$= \left(\frac{1}{x}\right)^2 + \frac{1}{x}$

g) $(f \circ f)(x) = f(f(x))$
$= f(2x-1)$
$= 2(2x-1) - 1$
$= 4x - 2 - 1$
$= 4x - 3$

h) $(f \circ g \circ h)(x) = f(g(h(x)))$

$= f\left(g\left(\frac{1}{x}\right)\right)$

$= f\left(\left(\frac{1}{x}\right)^2 + \frac{1}{x}\right)$

$= 2\left[\frac{1}{x^2} + \frac{1}{x}\right] - 1$

i) $(g \circ h \circ f)(x) = g(h(f(x)))$

$= g(h(2x-1))$

$= g\left(\frac{1}{2x-1}\right)$

$= \left(\frac{1}{2x-1}\right)^2 + \left(\frac{1}{2x-1}\right)$