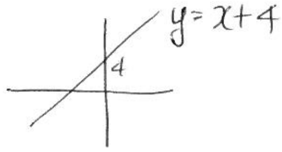
Functions - Refresher from Week 7

- Remember function machine concept

  eg: $\underset{\text{input}}{f(x)} = \underbrace{x+4}_{\text{output}}$

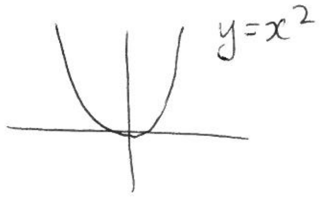
- Representing functions
  1. Algebraically
  2. Table of values
  3. Graphically



  4. Mapping $f: \mathbb{R} \to \mathbb{R}$ $\quad f(x) = x+4$

- Domain + Range.
  - Domain = all values you can put into function
  - Codomain (given) = values that <u>could</u> come out of function (usually a general set, type of number)
  - Range = values that <u>do</u> come out of function

eg: $f(x) = x^2$ $\quad f: \mathbb{R} \to \mathbb{R}$ ← codomain (given)



Dom $= \mathbb{R}$

Range $= [0, \infty) = \{y \in \mathbb{R} : y \geq 0\}$

- Finding domains → look for problems √☺ , 1/☺
- Finding ranges → best to use graph.

eg: $f(x) = \sqrt{x+4} + 1$

Domain: Need $x+4 \geqslant 0$

$x \geqslant -4$

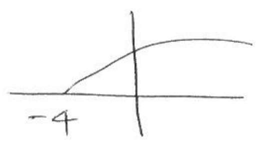
$\therefore$ Dom $= \{ x \in \mathbb{R} : x \geqslant -4 \}$

Range: First try and sketch this.

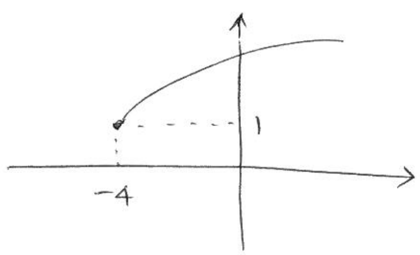
- Remember transformations on our standard curves.

Standard curve. $f(x) = \sqrt{x}$

$\therefore f(x) = \sqrt{x+4}$



$\therefore f(x) = \sqrt{x+4} + 1$



our graph confirms the domain $[-4, \infty)$

$\therefore$ Range $= \{ y \in \mathbb{R} : y \geqslant 1 \}$

- Recall modifying standard functions
  → vertical + horizontal shifts.

- Recall : The Circle

  eqn $(x-h)^2 + (y-k)^2 = r^2$ is the eqn of the circle with centre (h,k)
  radius r

· Sketching:
  - recognise standard functions
  - look for transformations + modifications
  - label intercepts : y intercept → Let $x=0$
    x intercept → Let $y=0$.

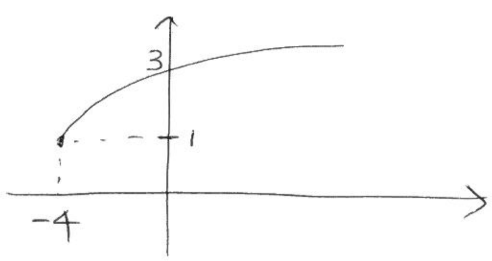
Back to eg: $f(x) = \sqrt{x+4} + 1$

y int → let $x=0$: $y = \sqrt{4} + 1 = 3$

x int → let $y=0$: $\sqrt{x+4} + 1 = 0$

$\sqrt{x+4} = -1$ ← No solution since $\sqrt{\ }$ gives only pos answers

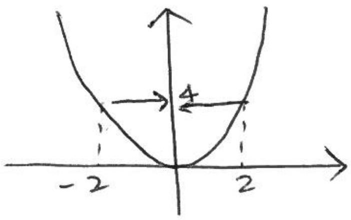
∴ No x intercept



Odd + Even Functions

Even Functions:

eg: $y = x^2$



Remember the special property of this graph $\rightarrow$ symmetrical about y axis

So $f(2) = f(-2)$

$f(3) = f(-3)$

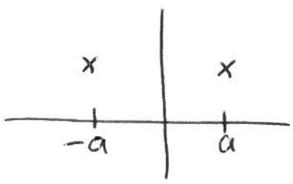
$f(\pi) = f(-\pi)$

ie: For every x in domain $f(x) = f(-x)$

We say $y = x^2$ is an even function

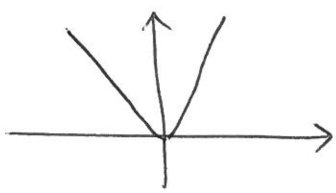
A function is even if $f(a) = f(-a)$ for every a in domain of f

ie: same thing happens at a and at $-a$



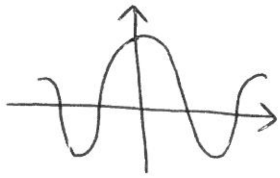
ie: Reflection about y axis

Other even functions:

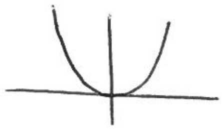


$y = |x|$

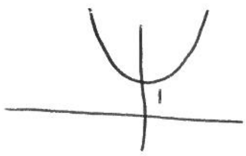
Look for symmetry about y axis



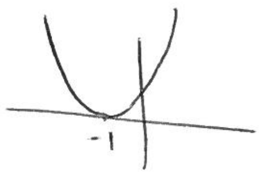
$y = x^2$ is even



$y = x^2 + 1$ is even



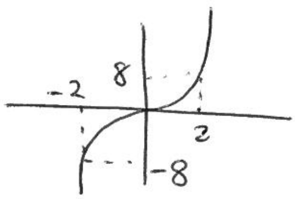
$y = (x+1)^2$ is NOT even



(Is Not symmetrical about y axis.)

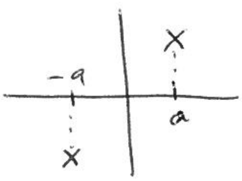
Odd Functions :

Now consider $y = x^3$



There is rotational symmetry here (180°)

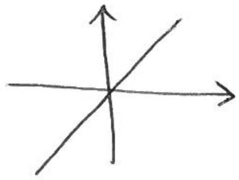
A function is odd if $f(-a) = -f(a)$ for every a in the domain of f



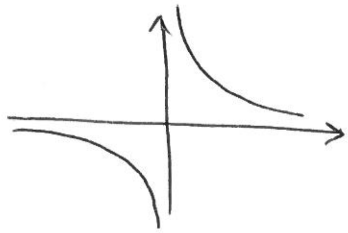
Rotation of 180° about the origin

So $y = x^3$ is an odd function.

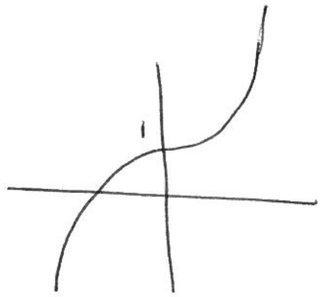
Also $y = x$ is odd.



So is $y = \frac{1}{x}$



$y = x^3 + 1$



is NOT odd.

(No rotational symmetry about the origin)

$f(x) = x^3 + 1$

check: $f(-2) = -8 + 1 = -7$

$-f(2) = -[8+1] = -9$

↑

we have found a counter example to prove this is not true.

- To prove something is not true we can find a counter example

- To prove something is true we have to show it for all values → use algebra.

eg: Show $f(x) = x^3$ is odd.

check $f(-2) = -8$ ✓

$-f(2) = -8$

But to prove it we do it in general.

$\therefore$ Let a = any value in domain of f.

$f(a) = a^3$ so $-f(a) = -a^3$

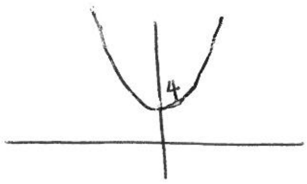
$f(-a) = (-a)^3 = -a^3$

$\therefore f(-a) = -f(a)$

$\therefore f(x)$ is an odd function.

Eg Sketch the functions + state whether they are even, odd or neither.

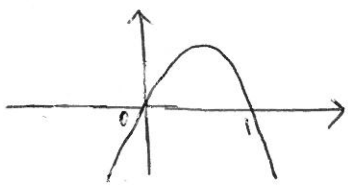
a) $f(x) = 3x^2+4$



Even (reflection about y axis)

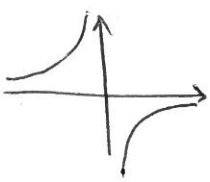
b) $f(x) = x - x^2$

$= x(1-x)$



Neither

c) $f(x) = \frac{-2}{x}$



Odd