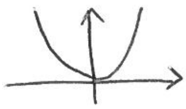
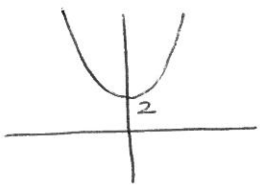
Modifying Functions

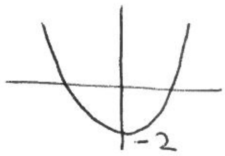
- Recall we did this when we looked at quadratics.

eg: Start with basic parabola $y = x^2$ 

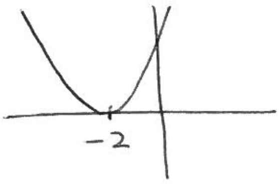
- $y = x^2 + 2$ ← vertical shift



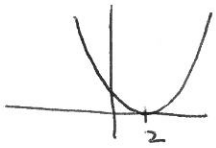
Also $y = x^2 - 2$



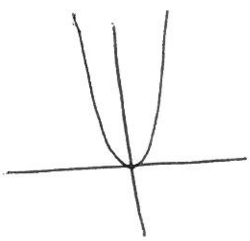
- $y = (x+2)^2$ ← horizontal shift



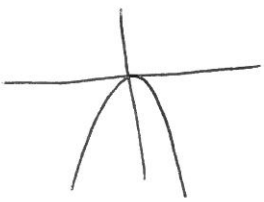
Also $y = (x-2)^2$



- $y = 2x^2$ ← affects steepness



- $y = -x^2$ ← negs become pos, pos becomes neg (ie: reflects about x axis)



- $y = \frac{1}{x^2}$ ← can't have 0 on bottom

$\frac{1}{\text{Big}}$ = small

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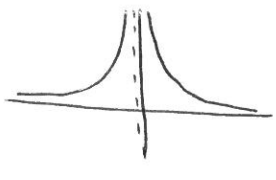


These transformations can be applied to any function

In general:

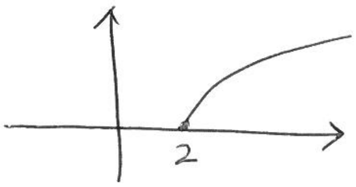
$y = f(x) \pm c$ ← vertical shifts

$y = f(x \pm a)$ ← horizontal shifts

$y = c\,f(x)$ ← affects steepness / stretch or shrink

eg: Recall we looked at $y = \sqrt{x-2}$.

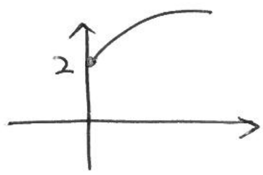
horiz. shift of sq root function



From graph we can see Dom = $[2, \infty)$
Range = $[0, \infty)$

eg: Sketch $f(x) = \sqrt{x} + 2$

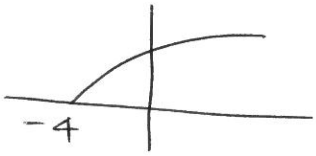
Now vertical shift



eg: Sketch $f(x) = \sqrt{x+4} - 5$

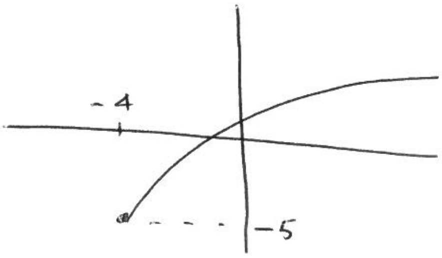
$f(x) = \sqrt{x}$

$f(x) = \sqrt{x+4}$



horiz shift

$f(x) = \sqrt{x+4} - 5$



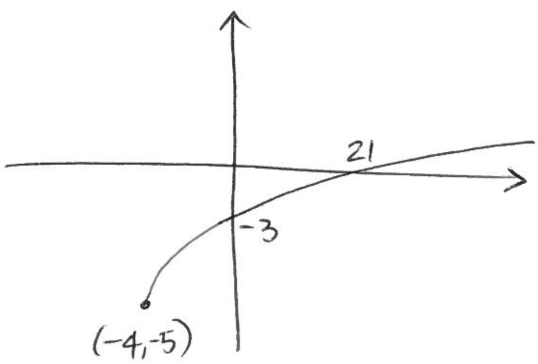
vertical shift down.

We can put in extra detals:

y int $\rightarrow$ Let $x=0$: $y = \sqrt{0+4} - 5$

$= \sqrt{4} - 5$

$= -3$



x int $\rightarrow$ Let $y=0$: $\sqrt{x+4} - 5 = 0$

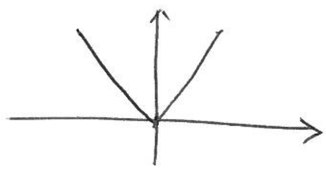
$\sqrt{x+4} = 5$

$x+4 = 25$

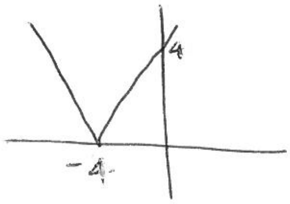
$x = 21$

eg Sketch $f(x) = 3|x+4| - 1$

Start with $y = |x|$

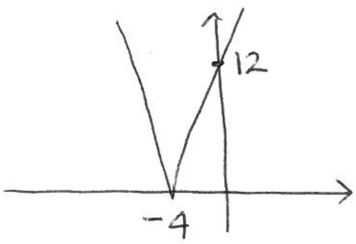


$y = |x+4|$ - shift left 4 units

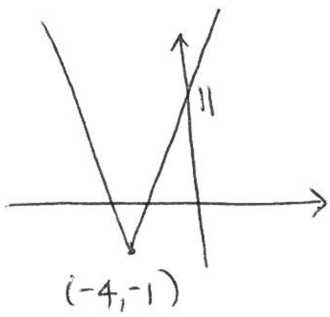


when $x=0$: $y = |0+4| = 4$

$y = 3|x+4|$ - made steeper ie: stretches.



$y = 3|x+4| - 1$ - vertical shift down by 1

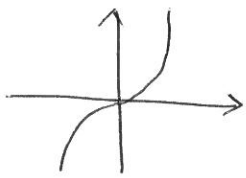


when $x=0$: $y = 3|0+4| - 1 = 11$

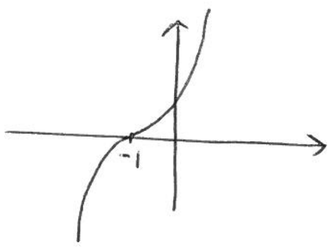
d) $f(x) = 2 - (x+1)^3$

$= -(x+1)^3 + 2$.

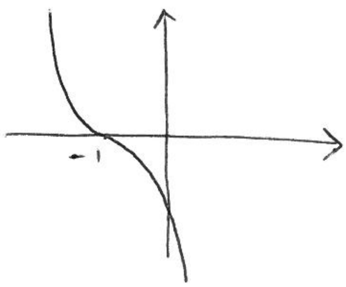
Start with $f(x) = x^3$



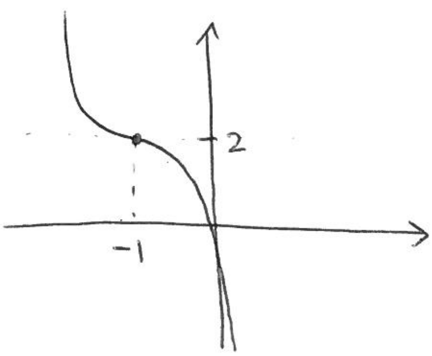
$f(x) = (x+1)^3$ horiz shift



$f(x) = -(x+1)^3$ – reflection in x axis



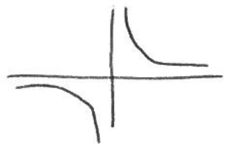
$f(x) = -(x+1)^3 + 2$ - vertical shift up 2



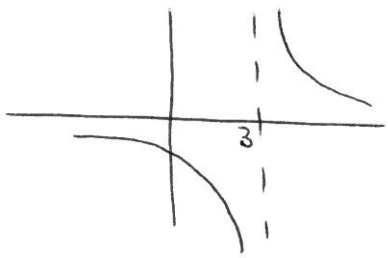
eg $f(x) = 1 + \dfrac{2}{x-3}$.

modifications of the hyperbola

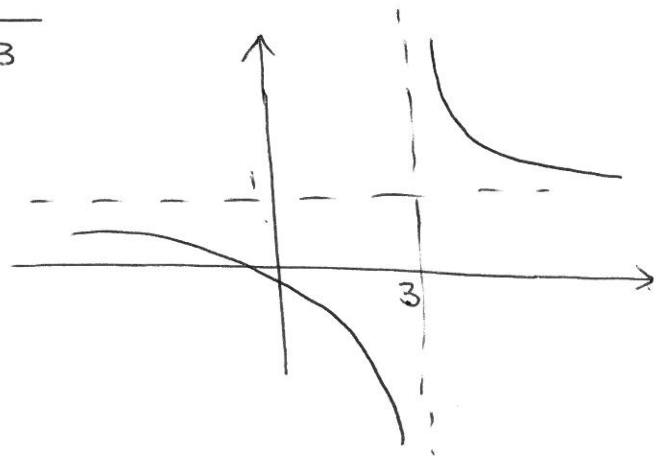
$\therefore f(x) = \dfrac{1}{x}$



$f(x) = \dfrac{2}{x-3}$



$f(x) = 1 + \dfrac{2}{x-3}$



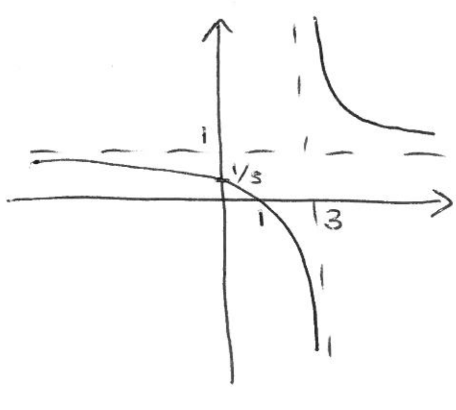
Note: we can find intercepts to see exactly where this sits

y int $\rightarrow$ Let $x = 0$: $y = 1 + \dfrac{2}{0-3} = \dfrac{1}{3}$

x int $\rightarrow$ Let $y = 0$: $0 = 1 + \dfrac{2}{x-3}$

$0 = x - 3 + 2$

$\therefore x = 1$



We can see: Dom $= \{x \mid x \neq 3\}$

Range $= \{y : y \neq 1\}$

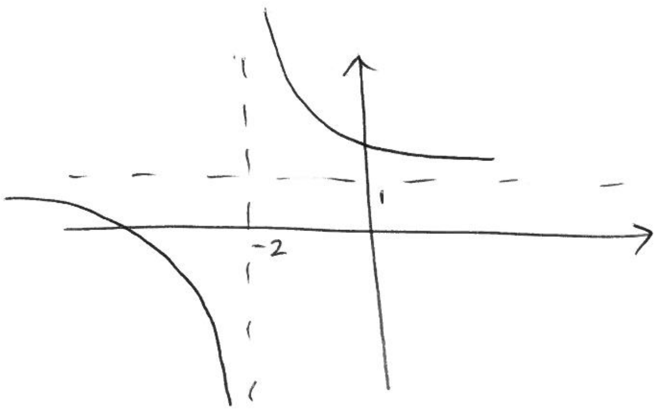
There is a trick we can do when we have a $\dfrac{\text{linear function}}{\text{linear function}}$

eg: $f(x) = \dfrac{x+5}{x+2}$

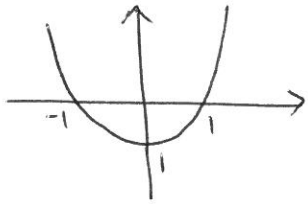
$= \dfrac{x+2+3}{x+2}$

$= \dfrac{x+2}{x+2} + \dfrac{3}{x+2}$

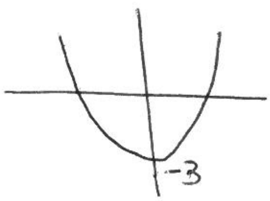
$= 1 + \dfrac{3}{x+2}$ ← Modified hyperbola

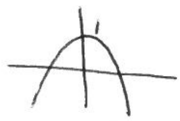


eg Let $f(x) = x^2 - 1$,

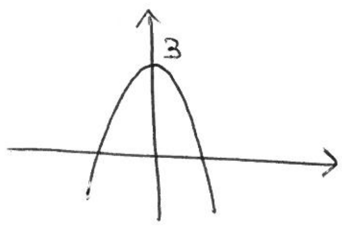
First draw $f(x) =$ 

(i) $f(x) - 2$:

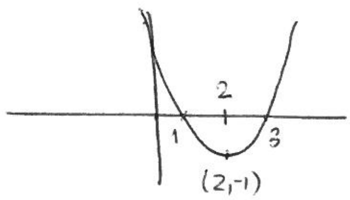


(ii) $2 - f(x)$: firstly $-f(x)$ looks like 

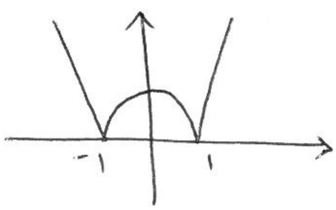
$\therefore$ $2 - f(x)$ shifts this up 2



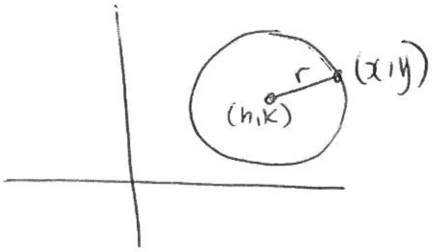
(iii) $f(x-2)$ - horizontal shift to right by 2



(iv) $|f(x)|$ - negatives become positive



The Circle



Think of circle as the set of points a given distance (r) from a given point (centre) (h,k)

So for any point on circle (x,y) its dist from (h,k) is r.

ie: dist formula $= \sqrt{(x-h)^2+(y-k)^2} = r$

ie: $\boxed{(x-h)^2+(y-k)^2 = r^2}$

↗

This is the eqn of a circle with centre (h,k) and radius r.

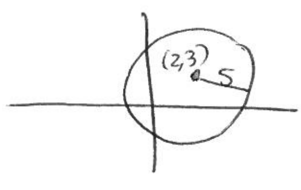
A circle centred at origin has eqn

$\boxed{x^2+y^2=r^2}$

Eg circle centre $(2,3)$ + radius 5 has eqn

$(x-2)^2+(y-3)^2 = 5^2$

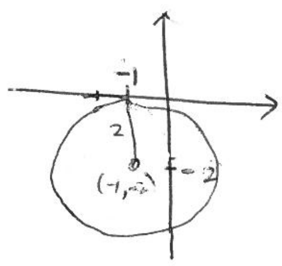
ie. $(x-2)^2+(y-3)^2 = 25$



circle centre $(-1,-2)$ and radius 2

$(x-(-1))^2+(y-(-2))^2 = (2)^2$

$(x+1)^2+(y+2)^2 = 4$



Eg Write down the centre + radius:

a) $(x-7)^2 + (y+5)^2 = 16$

ie: $(x-7)^2 + (y-(-5))^2 = 4^2$ $\therefore$ centre $= (7,-5)$ rad $= 4$.

b) $(x+1)^2 + y^2 = 3$

ie. $(x-(-1))^2 + (y-0)^2 = 3$ $\therefore$ centre $= (-1,0)$

radius $= \sqrt{3}$

c) $(x-1)^2 + (y-1)^2 = 1$ $\therefore$ centre $= (1,1)$

radius $= 1$.

d) $y^2 = 4 - x^2$

ie: $x^2 + y^2 = 4$ $\therefore$ centre $= (0,0)$

radius $= \sqrt{4} = 2$

e) $x^2 - 6x + y^2 = 10$ $\leftarrow$ Need $(x-h)^2 + (y-k)^2 = r^2$

$\therefore$ Complete square: $x^2 - 6x + y^2 = 10$

ie: $\underbrace{x^2 - 6x + 9} - 9 + y^2 = 10$

$(x-3)^2 - 9 + y^2 = 10$

$(x-3)^2 + y^2 = 19$

$\therefore$ Centre $= (3,0)$

Radius $= \sqrt{19}$

f) $x^2 + 10x + y^2 + 2y - 3 = 0$.

Need to complete square:

For the x's: $x^2 + 10x = x^2 + 10x + 25 - 25$

$= (x+5)^2 - 25$

For the ys: $y^2 + 2y = y^2 + 2y + 1 - 1$

$= (y+1)^2 - 1$

Putting them back we have

$x^2 + 10x + y^2 + 2y - 3 = 0$

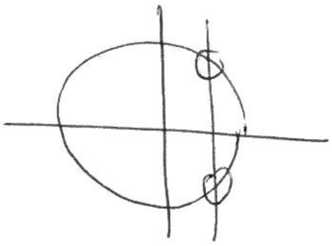
ie: $(x+5)^2 - 25 + (y+1)^2 - 1 - 3 = 0$

ie: $(x+5)^2 + (y+1)^2 = 29$

$\therefore$ centre $= (-5, -1)$

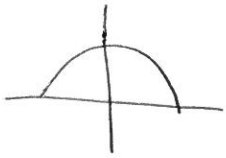
radius $= \sqrt{29}$

· Notice : A circle is not a funtion



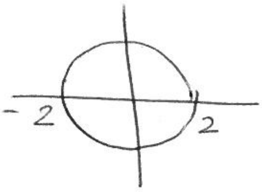
It fails the vertical line test.

But a semicircle is a function



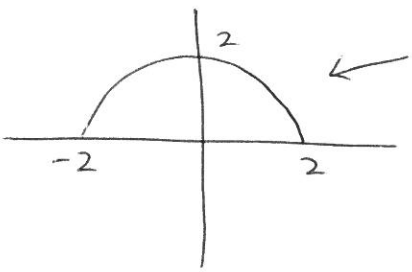
We can rearrange the equation of a circle to find the eqn of the semicircle.

eg:



$x^2+y^2=4$ is circle centered at $(0,0)$ + rad $=2$.

$$y^2=4-x^2$$
$$y=\pm\sqrt{4-x^2}$$

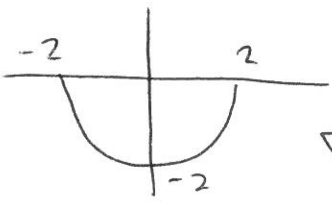


$\leftarrow y=+\sqrt{4-x^2}$

Dom $=[-2,2]$

Range $=[0,2]$

and



$\leftarrow y=-\sqrt{4-x^2}$

Dom $=[-2,2]$

Range $=[-2,0]$