Eg: Find the domain of the following functions.

a) $g(x) = \sqrt{x+5}$

Need $x+5 \geq 0$
$x \geq -5$

$\therefore$ Dom $= [-5, \infty) = \{ x \in \mathbb{R} : x \geq -5 \}$

b) $h(x) = \sqrt{2x-7}$

Need $2x-7 \geq 0$
$2x \geq 7$
$x \geq 7/2$ so Dom $= \{ x \in \mathbb{R} : x \geq \frac{7}{2} \}$

c) $f(x) = \dfrac{1}{x-3}$

Need $x-3 \neq 0$
ie: $x \neq 3$

$\therefore$ Dom $= \{ x \in \mathbb{R} : x \neq 3 \}$

d) $f(x) = \dfrac{1}{\sqrt{x-3}}$

Need $x-3 > 0$ (can't $=0$ either)
$x > 3$

$\therefore$ Dom $= (3, \infty) = \{ x \in \mathbb{R} : x > 3 \}$

e) $f(x) = \dfrac{1}{(x+1)(x+3)}$

Here $x \neq -1$ and $x \neq -3$

$\therefore$ Dom $= \{x \in \mathbb{R} : x \neq -1, x \neq -3\}$

f). $g(x) = \dfrac{1}{x^2+x-6}$

$= \dfrac{1}{(x+3)(x-2)}$

So $x \neq -3$, $x \neq 2$

Dom $= \{x \in \mathbb{R} : x \neq -3, x \neq 2\}$

g) $h(x) = \dfrac{x-2}{x^2-1}$

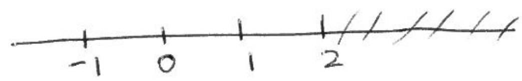
Bottom $\neq 0$ so $x \neq \pm 1$

$\therefore$ Dom $= \{x \in \mathbb{R} : x \neq \pm 1\}$

h) $h(x) = \dfrac{\sqrt{x-2}}{x^2-1}$

Need $x-2 \geqslant 0$ AND $x \neq \pm 1$

ie: $x \geqslant 2$ AND $x \neq \pm 1$




Since both need to hold

Dom $= [2, \infty)$

i) $f(x) = \dfrac{x+2}{x^2+1}$

The bottom is always $\geqslant 0$ regardless of $x$ value

$\therefore$ Dom $= \mathbb{R}$

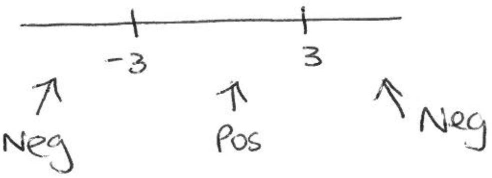
j) $g(x) = \sqrt{27-3x^2}$

Need $27 - 3x^2 \geqslant 0$

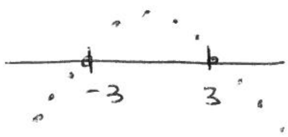
ie: $3(9-x^2) \geqslant 0$

ie: $9 - x^2 \geqslant 0$

Notice when $x = \pm 3$, $9 - x^2 = 0$




OR



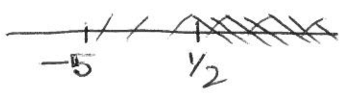

We want $9 - x^2 \geqslant 0$ so take $-3 \leqslant x \leqslant 3$

$\therefore$ Dom $= [-3, 3]$

k) $f(x) = \sqrt{2x-1} + \sqrt{x+5}$

Need both $2x - 1 \geqslant 0$ AND $x + 5 \geqslant 0$

ie: $x \geqslant \frac{1}{2}$ $\qquad x \geqslant -5$



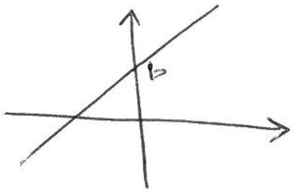

Both work for $x \geqslant \frac{1}{2}$

$\therefore$ Dom $= \{ x \in \mathbb{R} : x \geqslant \frac{1}{2} \}$

# Common Functions

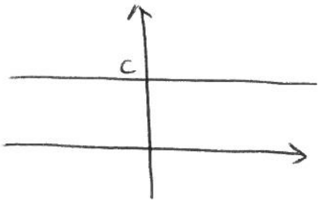
1. Linear Functions
ie: Lines.

$y = mx + b$



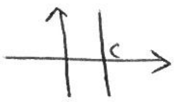

Dom = $\mathbb{R}$
Range = $\mathbb{R}$

$y = c$ (constant function)
(eg: $y=5$)



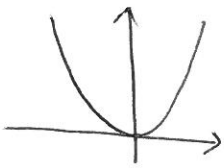

Dom = $\mathbb{R}$
Range = $\{c\}$

Note: $x = c$ IS NOT a function



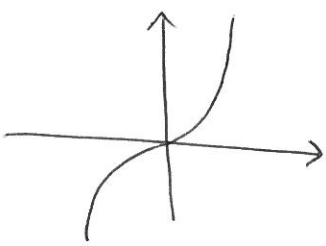

2. Power functions.

- Quadratic : $y = x^2$
ie: Parabolas



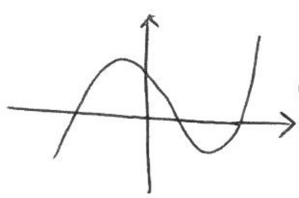
Dom = $\mathbb{R}$
Range = $[0,\infty)$

- Cubic : $y = x^3$

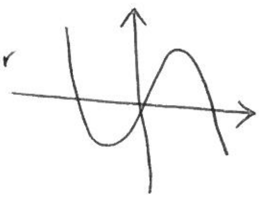


Dom = $\mathbb{R}$
Range = $\mathbb{R}$

[ or $y = ax^3 + bx^2 + cx + d$



or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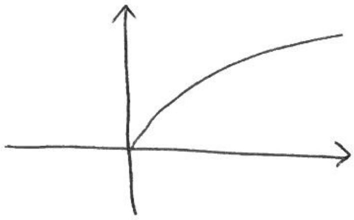
3. Root Functions

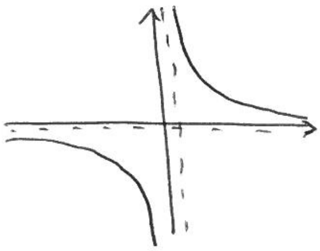
$y = \sqrt{x}$



Dom = $[0, \infty)$
Range = $[0, \infty)$

4. Recriprocal Functions

$y = \frac{1}{x}$



Dom = $\{x \mid x \neq 0\}$
Range = $\{y \mid y \neq 0\}$

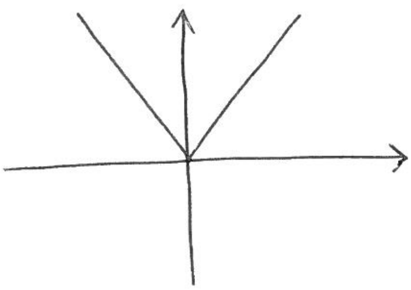
Rectangular Hyperbola

5. Absolute Value Function

$y = |x|$

$= \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases}$

Recall definition



Dom = $\mathbb{R}$
Range = $(0, \infty)$

6. Piecewise (or Split) Function

eg: $y = |x| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases}$ is a piecewise function

↑ defined by different formulas in different sections of domain

eg: $f(x) = \begin{cases} 1 + x & \text{if } x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$ ←