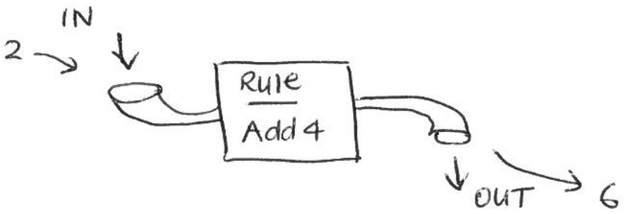
Functions

- A function is a rule.
- Function machine:



Input	2	-1	0	x
Output	6	3	4	$x+4$

We say

$$f(2) = 6$$
$$f(-1) = 3$$
$$f(0) = 4$$
$$\underset{\text{input}}{f(x)} = \underbrace{x+4}_{\text{output}}$$

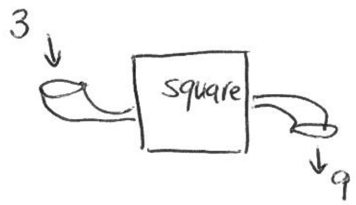
- Functional notation
- x represents the input, not the letter x.
- Learn to describe functions using words.

eg $f(x) = x+4$ → Add 4 to input

$f(x) = x^2$ → square the input

so $f(3) = 3^2 = 9$

$f(7) = 7^2 = 49$



Eg

a) $f(x) = x^2 + 1$ ← square, then add 1

b) $f(x) = (x+1)^2$ ← add 1, then square

c) $f(x) = x^2 + x$ ← square input then add to itself

same as $f(t) = t^2 + t$

↑ input

- x, t are telling us what to do with input

Eg2) $f(t) = t^2 + t$

To Evaluate $f(-2)$ we apply this rule to input -2.

a) ie: $f(-2) = (-2)^2 + (-2)$
$= 4 - 2$
$= 2$

b) $f(0) = 0^2 + 0 = 0$

c) $f(\frac{1}{2}) = (\frac{1}{2})^2 + (\frac{1}{2}) = \frac{1}{4} + \frac{1}{2}$
$= \frac{3}{4}$

↑

We have been <u>substituting</u> values into our function

Ways of Representing Functions

1. Algebraic Expression (Function Notation) : $f(x) = x + 4$

$g(x) = x+4$ tells us the same thing

$g(t) = t+4$ " " " " "

g is a function of x

g is a function of t

2. Table of Values

x	0	1	2
$f(x)$	4	5	6

we get specific inputs + output here.
so we can think of this graphically.

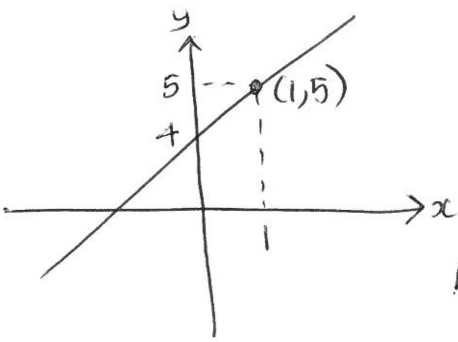
3. Graphically
   - we plot points (x, y)

input output

we say $y = f(x) = x + 4$ and

x	0	1	2
y	4	5	6

points on graph



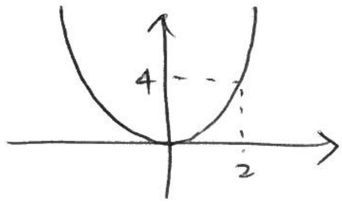
can see inputs on x axis + corresponding outputs on y axis

- recognise this as the line $y = x + 4$

or $f(x) = x + 4$

- Lines are functions
- Quadratics are functions

$f(x) = x^2$



when we write $y = f(x)$ we say

"y is a function of x"

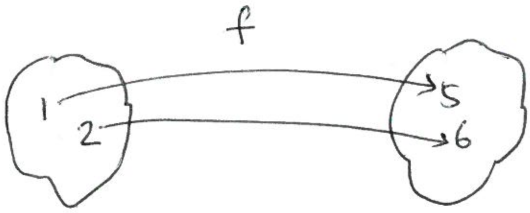
"y is determined by x"

"y depends on x"

we say x = independent variable

y = dependent variable

4. As a mapping



$f(x) = x + 4$

set of inputs — set A

outputs — set B

f takes values in one set and "maps" it to values in the other.

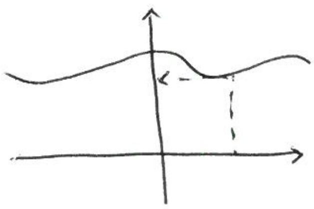
We say $f : A \to B$

Definition of a function

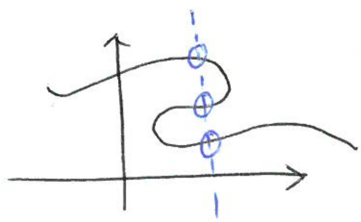
A function is a rule that assigns each element in a set A, to exactly one other element in set B.

ie: For every input, we get a single output

Graphically:



This is a function



This is NOT a function

use "vertical line test" to see this.

Doman + Range

Domain = all possible values we can put into a function
ie: set of inputs (x-values)

Range = all the values that come out of a function
ie: set of outputs (y-values)

- think of a function as a mapping when we talk about domain + range.

We write: $f: A \to B$

A ↑ Domain

B ↑ contains range ← codomain (given)

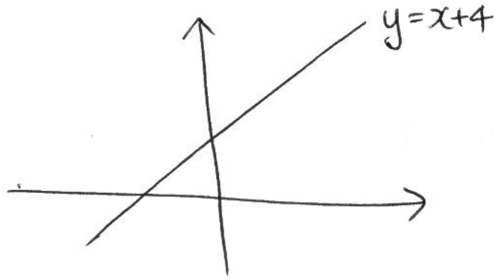
eg: $f: \mathbb{R} \to \mathbb{R}$ where $f(x) = x+4$

↑ Here f takes real n°s + maps them to real n°s.

Domain = $\mathbb{R}$

Codomain = $\mathbb{R}$

We also know Range = $\mathbb{R}$
+ we can see this from the graph



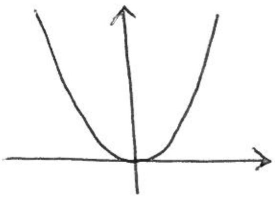
note: . $f: D \to \mathbb{R}$ we say f is a real valued function
(since its output is real n°s)

So domain = values we start from

codomain = everything we could go to

range = everything we do go to.

eg: $f: \mathbb{R} \to \mathbb{R}$ $\quad f(x) = x^2$



Dom $= \mathbb{R}$

↑ every real n° can be put into this function

But notice the outputs are ≥ 0

No neg numbers come out.

$\therefore$ Range $= [0, \infty)$ ← interval notation

$= \{ y \in \mathbb{R} : y \geq 0 \}$

↗ Remember set notation

- Finding Domains → Look out for problems.

eg: $\sqrt{\quad}$ $\qquad \dfrac{1}{\quad}$

↑ cant be neg $\qquad$ ↖ can't be 0

- Finding Ranges → depends on what we put in (domain)

→ graph is often helpful

eg Find the domain + range of $f(x) = \sqrt{x-2}$

Domain: Need $x - 2 \geq 0$

$x \geq 2$

$\therefore$ Dom $= [2, \infty) = \{x \in \mathbb{R} : x \geq 2\}$

Range: Start with values we can put in

when $x = 2$, $f(x) = \sqrt{0} = 0$

$x = 3$ $f(x) = \sqrt{1} = 1$

$x = 4$ $f(x) = \sqrt{2} = 1.4$

$\vdots$ $\vdots$

$\downarrow$ $\downarrow$

as x increases — y values increase (slowly)

- the smallest y will be is 0
- then will keep increasing

$\therefore$ Range $= \{y \in \mathbb{R} : y \geq 0\}$

Infact the graph of this looks like: