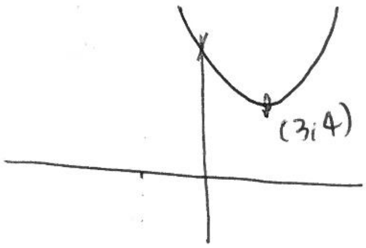
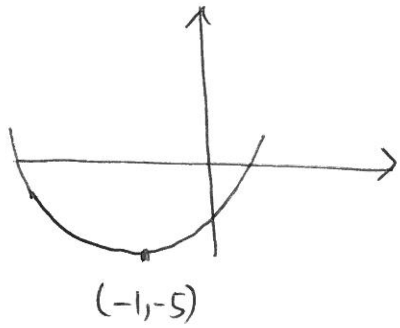
g) $y = (x-3)^2 + 4$



- facing up
- vertex (3,4)

h) $y = \frac{1}{2}(x+1)^2 - 5$



- facing up
- vertex (-1,-5)

Notice $y = (x-3)^2 + 4$ ← vertex form

$= (x^2 - 6x + 9) + 4$

$= x^2 - 6x + 13$ ← standard form

- All quadratics can be written in vertex form.

Notice what happens: $y = x^2 - 6x + 13$

$= \underbrace{x^2 - 6x + 9}_{\text{perfect square}} + 4$

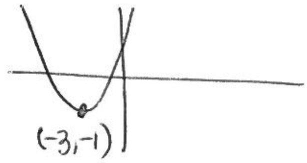
$= (x+3)^2 + 4.$

what about x^2+6x+8 ?

we know $x^2+6x+9 = (x+3)^2$ (Perf Sq Rule)

So $x^2+6x+8 = x^2+6x+9-1$

$= (x+3)^2-1$



↗

we have just completed the square.

ie: $y = x^2+6x+8$ ← standard form

$= \underbrace{x^2+6x+9}-9+8$

$= (x+3)^2-1$ ← vertex form

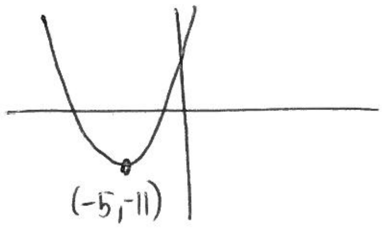
vertex $= (-3,-1)$

Eg2) Complete the square + sketch the parabolas.

a) $y = x^2+10x+14$

$= x^2+10x+25-25+14$

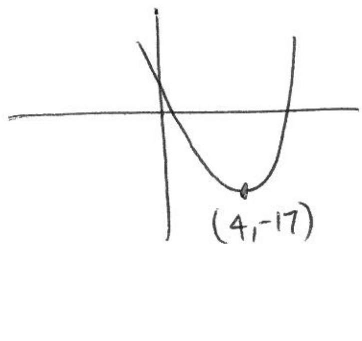
$= (x+5)^2-11$



b) $y = x^2-8x-1$

$= x^2 \underset{\substack{\shortparallel \\ 2(4)}}{-8x} + \underset{\substack{\shortparallel \\ (4)^2}}{16} -16-1$

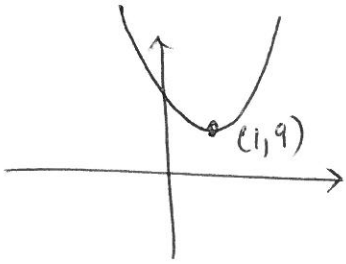
$= (x-4)^2-17$



c) $y = x^2 - 2x + 10$

$= x^2 - 2x + 1 - 1 + 10$

$= (x-1)^2 + 9$



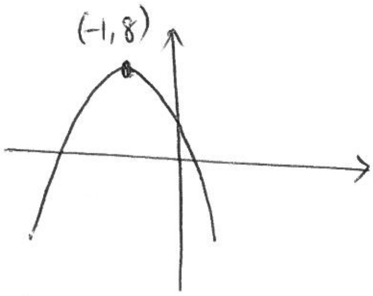
d) $y = -x^2 - 2x + 7$

$= -\left[x^2 + 2x - 7\right]$

$= -\left[x^2 + 2x + 1 - 1 - 7\right]$

$= -\left[(x+1)^2 - 8\right]$

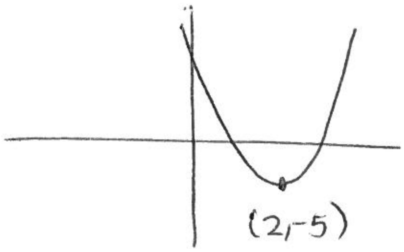
$= 8 - (x+1)^2$



e) $y = x^2 - 4x - 1$

$= x^2 - 4x + 4 - 4 - 1$

$= (x-2)^2 - 5$



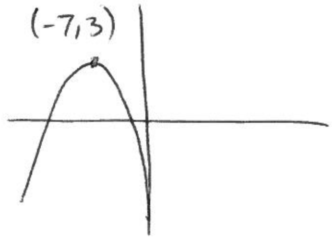
f) $y = -x^2 - 14x - 46$

$= -\left[x^2 + 14x + 46\right]$

$= -\left[x^2 + 14x + 49 - 49 + 46\right]$

$= -\left[(x+7)^2 - 3\right]$

$= 3 - (x+7)^2$



9) $y = 2x^2 + 12x - 1$

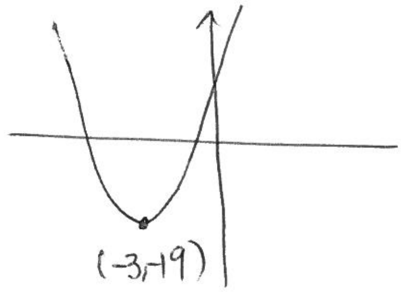
$= 2\left[x^2 + 6x\right] - 1$

$= 2\left[\underbrace{x^2 + 6x + 9} - 9\right] - 1$

$= 2\left[(x+3)^2 - 9\right] - 1$

$= 2(x+3)^2 - 18 - 1$

$= 2(x+3)^2 - 19$

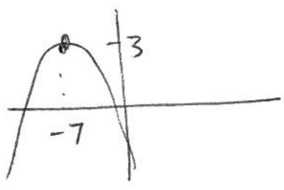


Notice : The vertex gives us max + min values.

eg (above) $y = 2(x+3)^2 - 19$

Here the min value is -19 and this occurs when $x = -3$

eg3) $y = 3 - (x+7)^2$



Here we have a max value $= 3$

This occurs when $x = -7$.

eg 4) Find the min value of $y = x^2 - 6x + 2$

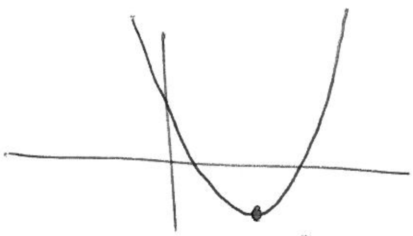
Note: this is not in vertex form so we can't see it

$\therefore$ First write in vertex form.

complete sq: $y = x^2 - 6x + 2$

$= x^2 - 6x + 9 - 9 + 2$

$= (x-3)^2 - 7$



$\therefore$ Min value is -1

+ this occurs when $x = 3$

Solving Quadratic Equations

We want to solve equations of the form

$$ax^2+bx+c=0$$

equation

Recall: Solve = find x that makes this statement true

Aim – Find x.

① Solve by Factorising

- We've looked at this before
- Remember your factorising techniques

  Diff of 2 Sq
  Perf Sq Rules
  Product - Sum method.

eg 5) a) $x^2-25=0$ (Diff of 2 sq)

$$(x+5)(x-5)=0$$

$$x=\pm 5$$

b) $\underset{\substack{\parallel \\ 2(4)}}{x^2-8x}+\underset{\substack{\parallel \\ (4)^2}}{16}=0$ (Perf Sq Rules)

$$\therefore (x-4)^2=0$$

$$x=4$$

c) $x^2+4x+3=0$ (Prod-Sum)

$$(x+3)(x+1)=0$$

$$x=-3,-1$$

② Solve by taking sq roots.

eg6a) $x^2 - 25 = 0$ can be solved another way

ie: $x^2 = 25$

We know $5^2 = 25$, Also $(-5)^2 = 25$

So $x = \pm 5$

Note: This ques is diff to . What is $\sqrt{25} = +5$

↑ only pos.

eg b) Solve $x^2 = 7$

$\therefore x = \pm\sqrt{7}$

eg c) Solve $(x-1)^2 - 49 = 0$ ← when in vertex form this method of solving works well.

ie: $(x-1)^2 = 49$

$x - 1 = \pm 7$

$x - 1 = 7$ $\quad$ $x - 1 = -7$

$x = 8$ $\quad$ $x = -6$

eg d) Solve $5 - 2(x+3)^2 = 0$

ie: $2(x+3)^2 = 5$

$(x+3)^2 = 5/2$

$x + 3 = \pm\sqrt{5/2}$

$\therefore x = 3 \pm \sqrt{5/2}$