# Quadratics

- We have already met quadratic expressions $ax^2+bx+c$
- The equation of a quadratic is of the form

$$y=ax^2+bx+c$$

where $a, b, c$ = constants $a \neq 0$ (we say degree = 2)

- We need to recognise quadratics.

$y = 4x^2 - 3x + 5$ ← here $a=4$ $b=-3$ $c=5$

↑ coefficient of $x^2$ ↑ coeff of $x$ ↑ constant term

$y = x^3 + \frac{1}{x^2}$ is NOT a quadratic

$y = x^2 - 2\sqrt{x} + 1$ is NOT a quadratic

- Recall $y = ax^2+bx+c$

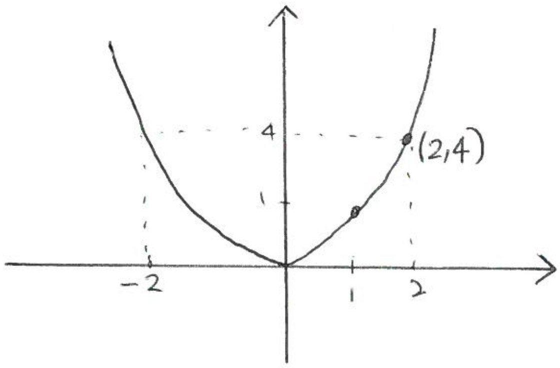
  ↖ Relationship between $x$ and $y$

  - Eqn which tells us how to get $y$ value from $x$ value gives us some sort of shape
  - Quadratic eqn gives us a particular shape.

Basic quadratic : $y = x^2$ (ie: $a=1, b=0, c=0$)

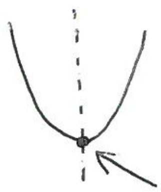
Sketch

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $9$ | $4$ | $1$ | $0$ | $1$ | $4$ | $9$ |



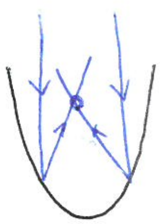

. This shape is called a parabola
. All quadratics give us parabolas

- what's special about a parabola?



symmetric about vertical line (axis of symmetry)

vertex = where line of symmetry intersects parabola



It's a reflector
- any ray parallel to axis of symmetry gets reflected straight to focus

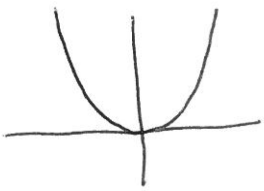
∴ used for
- satalite dishes
- radar dishes
- reflectors on torches

etc.

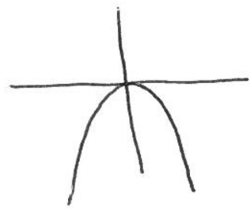
- The graph of a quadratic is a parabola


- A parabola that faces up will always face up ← $a>0$
- " " " " down " " " down ← $a<0$

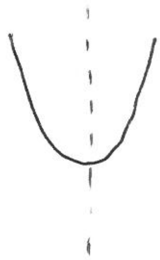
$y = x^2$



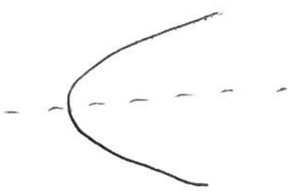
$y = -x^2$



- Every parabola has an axis of symmetry

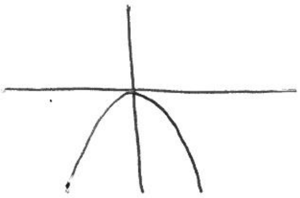


mirror image on other side



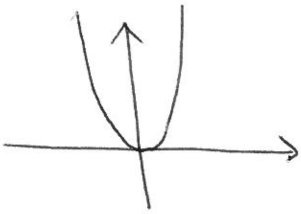
## Modifying the basic parabola

$y = -x^2$



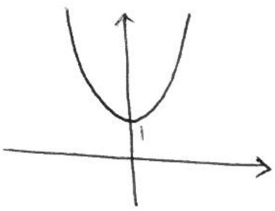
- vertex $= (0,0)$
- pos beiome neg

$y = 2x^2$



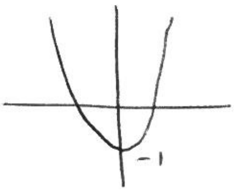
- vertex $(0,0)$
- steeper

$y = x^2 + 1$



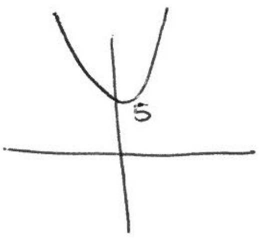

- vertex $(0,1)$
- vertical shift

$y = x^2 - 1$



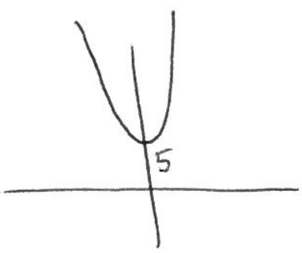

vertex $(0,-1)$

$y = x^2 + 5$



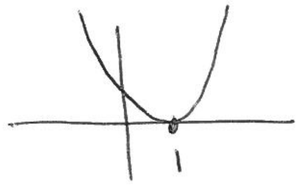

vertex $(0,5)$

$y = 2x^2 + 5$



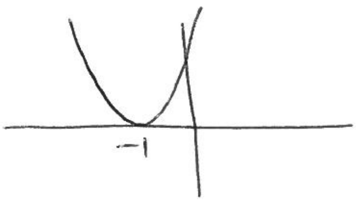

vertex $(0,5)$

$y = (x-1)^2$



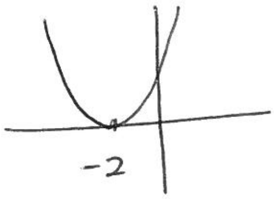

- horiz shift
- when $x=1$, $y=0$
- vertex = $(1,0)$

$y = (x+1)^2$




- horiz shift to left
- vertex $(-1,0)$

$y = (x+2)^2$

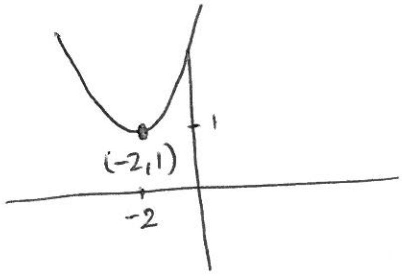



vertex $(-2,0)$

Eg 1 a) $y = (x+2)^2 + 1$

$(x+2)^2$ ↑ horiz shift

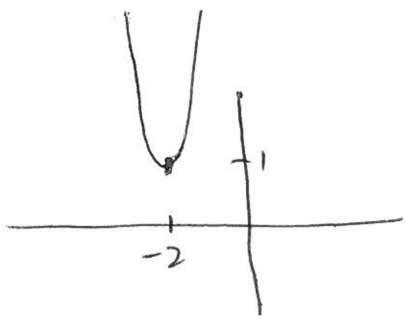
$+1$ ↑ vertical shift




vertex = $(-2,1)$

b) $y = 3(x+2)^2 + 1$

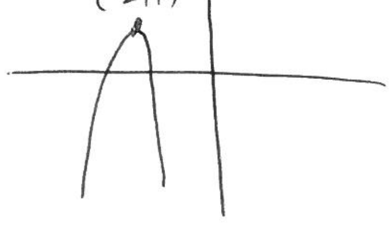
$3$ ↑ affects steepness




vertex $(-2,1)$

c) $y = -3(x+2)^2 + 1$

$-$ ↑ now facing down




vertex $(-2,1)$

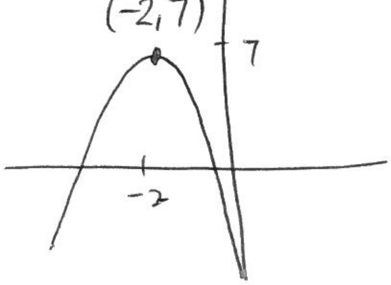
d) $y = 7 - (x+2)^2$

$= -(x+2)^2 + 7$

facing down

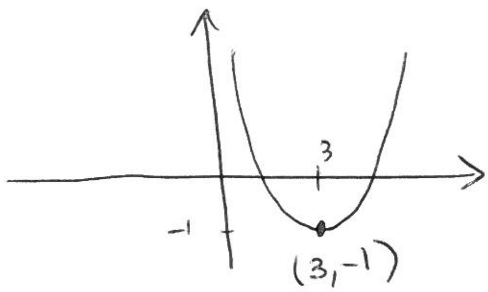
horiz shift to $-2$

vertical shift up 7



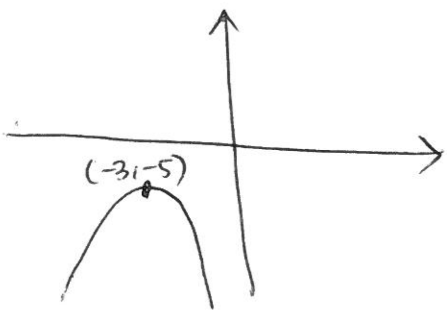

vertex $= (-2,7)$

e) $y = 2(x-3)^2 - 1$




- facing up
- vertex $= (3,-1)$

f) $y = -2(x+3)^2 - 5$




- facing down
- vertex

So we actually have another way of writing our parabola equation:

$$\boxed{y = a(x-h)^2 + k} \leftarrow \text{Vertex Form}$$

Notice: • $(h,k)$ = vertex

- If $a > 0 \rightarrow$ parabola faces up

  $a < 0 \rightarrow$ parabola faces down