Eg 6) Find the $x + y$ intercepts + sketch the lines

a) $3x - 5y + 30 = 0$

$x$ int → Let $y=0$: $3x + 30 = 0$

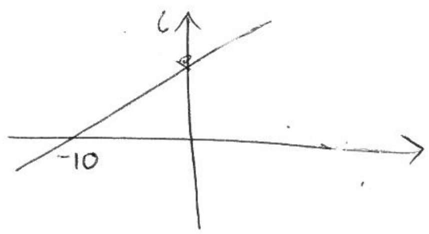
$3x = -30$

$x = -10$

$y$ int → Let $x=0$: $-5y + 30 = 0$

$-5y = -30$

$y = 6$



(check: $-5y = -3x - 30$

$y = \frac{3}{5}x + 6$)

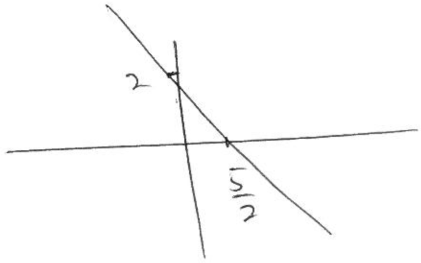
b) $4x + 5y = 10$

$x$ int → Let $y=0$: $4x = 10$

$x = \frac{10}{4} = \frac{5}{2}$

$y$ int → Let $x=0$: $5y = 10$

$y = 2$

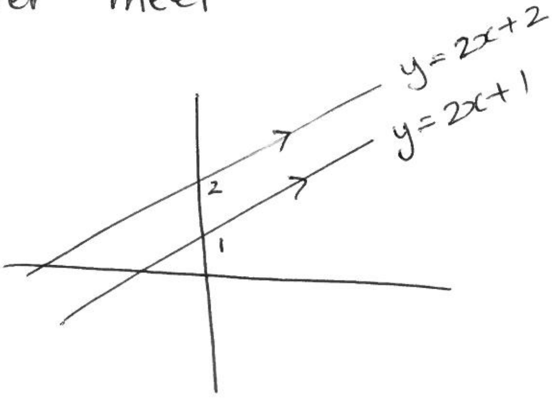


(check: $5y = -4x + 10$

$y = -\frac{4}{5}x + 2$)

# Parallel Lines

never meet




- These lines have the same slope (the y-int tells us where they sit).
- Notice they are parallel
- Notice they will never meet

$$\boxed{\text{Parallel lines have the same gradient}}$$

eg: $y=2x+1$ and $y=2x+2$ are parallel

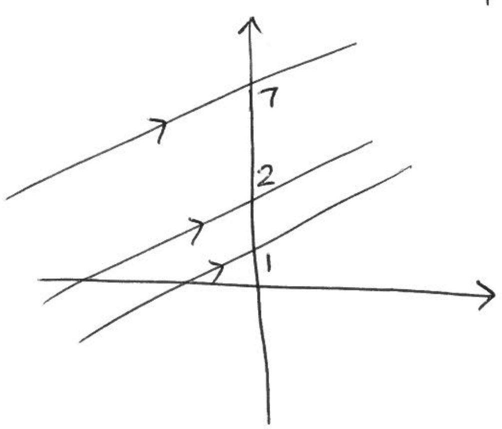
What about the line $10x-5y+35=0$

Rearrange: $5y = 10x+35$

$y = 2x+7$

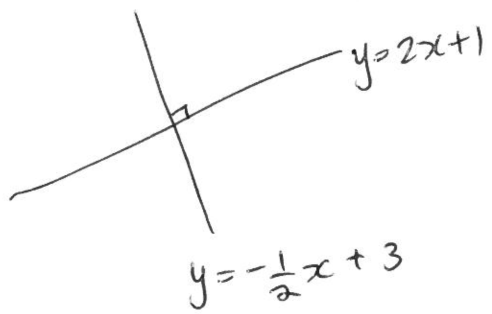
This also has $m=2$

so this is parallel to the others



## Perpendicular Lines

= meet at right angles




$\boxed{\text{When two lines are perpendicular the product of their gradients is } -1}$

ie: $m_1 \times m_2 = -1$

eg) $y = 2x + 1$

Perp line has slope $m$ where $m \times 2 = -1$

$\therefore m = -\frac{1}{2}$

2) $y = -3x + 7$

Perp line has slope $m$ where $m \times -3 = -1$

$m = \frac{1}{3}$

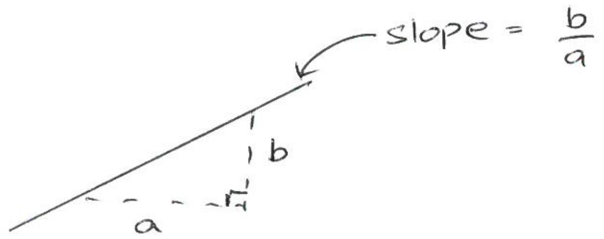
3) Slope perpendicular to $m_1 = \frac{5}{4}$ is $m_2$ : $m_2 \times \frac{5}{4} = -1$

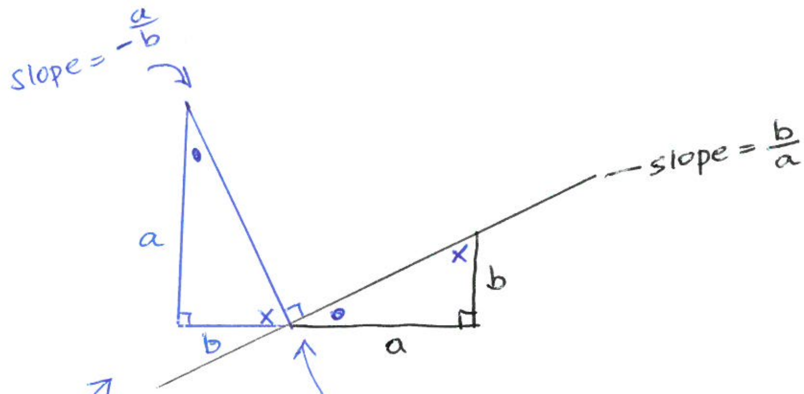
$m_2 = -\frac{4}{5}$

Notice we flip + put neg sign on

4) Slope perpendicular to $m_1 = -\frac{7}{8}$ is $m_2 = \frac{8}{7}$

# Perpendicular Lines – Proof




- Rotate triangle
- Notice angles.
- We have a right angle in the middle
- Notice slopes:

$$\frac{a}{b} \times -\frac{b}{a} = -1$$

ie: When 2 lines are perpendicular their slopes multiply to give $-1$

eg7) Find the eqn of the line passing through (5,3) + parallel to $2x+5y=7$.

Line: $y-y_1=m(x-x_1)$ + $(x_1,y_1)=(5,3)$

Finding m: parallel to $2x+5y=7$

$$5y=-2x+7$$

$$y=-\frac{2}{5}x+7/5$$

$\therefore m=-\frac{2}{5}$ since parallel lines have same slope

$$\therefore y-3=-\frac{2}{5}(x-5)$$

ie: $y-3=-\frac{2}{5}x+2$

$$y=-\frac{2}{5}x+5$$

eg8) Eqn of line passing through (5,3) + perpendicular to $2x+5y=7$.

Line: $y-y_1=m(x-x_1)$

This time $m\times-\frac{2}{5}=-1$

$$\therefore m=\frac{5}{2}$$

$$\therefore y-3=\frac{5}{2}(x-5)$$

$$y-3=\frac{5}{2}x-\frac{25}{2}$$

$$y=\frac{5}{2}x-\frac{25}{2}+3$$

$$y=\frac{5}{2}x-\frac{19}{2}$$

We say this line is normal to $2x+5y=7$.

(* Try Q9 - Problem Set)

## Point of Intersection

= where 2 lines meet (ie: intersect)

eg10) a) Consider the two lines $y = 2x - 1$
$y = 3x + 3$

Their slopes are different
$\therefore$ Not parallel $\therefore$ They must meet.

$\therefore$ Find their point of intersection $\rightarrow$ solve simultaneously.

ie: $2x - 1 = 3x + 3$

$-x = 4$

$x = -4$

So when $x = -4$ : $y = 2(-4) - 1 = -9$

$\therefore$ Point of intersection is $(-4, -9)$

↗
We have just solved simultaneous equations.
(by substitution)

eg b) Lines $x + 2y = 3$

$2x - y = 6$

Solving simultaneously

$x + 2y = 3$ — ①

$2x - y = 6$ — ②

Method 2: By elimination : ① × 2 : $2x + 4y = 6$ — ③

③ − ② $5y = 0$ (x's eliminated)

$\therefore y = 0$

$\therefore$ sub back in ①: $x = 3 - 2y$

$= 3$

$\therefore$ point of int is $(3, 0)$

eg c) $x + 2y = 3 \rightarrow y = -\frac{1}{2}x + \frac{3}{2}$

$2x + 4y = 1 \rightarrow 4y = -2x + 1$

$y = -\frac{1}{2}x + \frac{1}{4}$

These lines are parallel so they will never meet

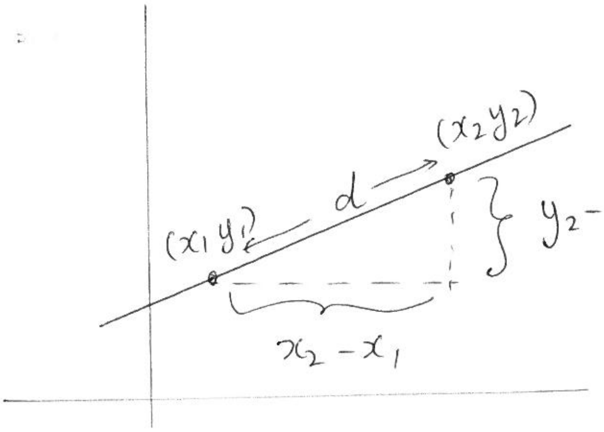
No point of intersection.

eg d) $x + 2y = 3$

$2x + 4y = 6$ ← These are the same line

$\therefore$ They have infinite points of intersection

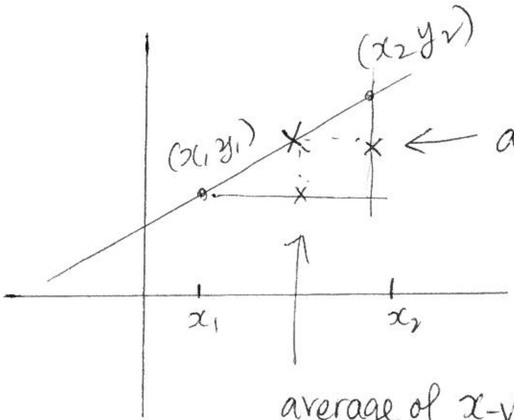
# Distance + Midpoint Formulas




By Pythagoras $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$$\therefore \boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

= distance between 2 points.




← average of y-values $= \dfrac{y_1 + y_2}{2}$

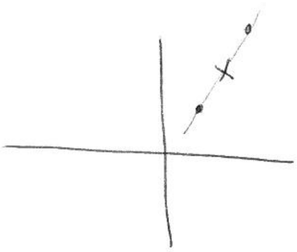
average of x-values $= \dfrac{x_1 + x_2}{2}$

$$\therefore \boxed{\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

eg) consider points $(1, 3)$ and $(7, 10)$

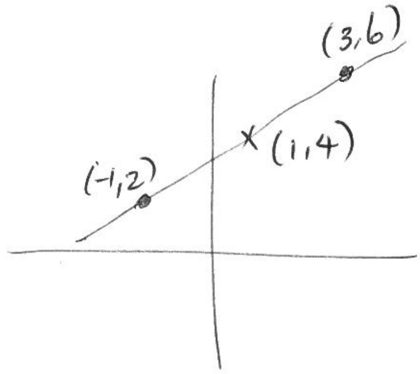
Dist between points $= \sqrt{(7-1)^2 + (10-3)^2} = \sqrt{6^2 + 7^2} = \sqrt{91}$

Midpoint $= \left(\frac{7+1}{2}, \frac{10+3}{2}\right) = \left(4, \frac{13}{2}\right)$



b) $(-1,2)$ and $(3,6)$

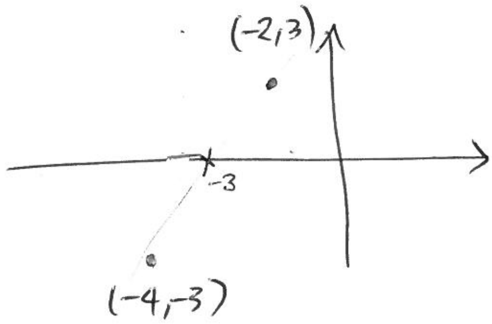
$$\text{dist} = \sqrt{(3-(-1))^2+(6-2)^2}$$
$$= \sqrt{4^2+4^2}$$
$$= \sqrt{32}$$




$$\text{midpt} = \left(\frac{-1+3}{2}, \frac{2+6}{2}\right)$$
$$= (1, 4)$$

c) $(-2,3)$ and $(-4,-3)$

$$\text{dist} = \sqrt{(-4-(-2))^2+(-3-3)^2}$$
$$= \sqrt{(-2)^2+(-6)^2}$$
$$= \sqrt{4+36}$$
$$= \sqrt{40}$$




$$\text{midpt} = \left(\frac{-2-4}{2}, \frac{3-3}{2}\right)$$
$$= (-3, 0)$$

eg) Find all the points $(4, y)$ that are 10 units from $(-2, -1)$

ie: distance between $(4, y)$ and $(-2, -1)$ is 10

So $\quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ie: $\quad 10 = \sqrt{(4-(-2))^2 + (y-(-1))^2}$

ie: $\quad 100 = (6)^2 + (y+1)^2$

ie: $\quad 100 = 36 + y^2 + 2y + 1$

ie: $\quad y^2 + 2y + 37 - 100 = 0$

$y^2 + 2y - 63 = 0$ — $\left.\begin{array}{l}\text{Prod} = -63 \\ \text{sum} = 2\end{array}\right\} -7, 9$

$\therefore (y-7)(y+9) = 0$

$\therefore y = 7, -9$

$\therefore$ The points are $(4, 7)$ and $(4, -9)$

# Linear Models

$$y = mx + b$$

gradient $= \dfrac{\Delta y}{\Delta x}$ = rate of change

eg: $y = 2x - 1$

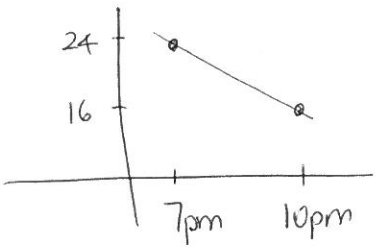
| $x$ | $y$ |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 3 | 5 |
| 4 | 7 |

2
2

for every unit change in $x$
$y$ changes by 2 units

Recognise linear model by constant rate of change.

13. At 7pm the temperature is 24 degrees Celcius. At 10pm the temperature has dropped to 16 degrees Celcius. Find the average rate of change in the temperature.




← Temp is decreasing
∴ Slope is neg.

$$\text{Rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{24-16}{7-10} = -\frac{8}{3}$$

$$= 2.66$$

ie: Temp is decreasing by 2.66°C per hour.

14. Scientists believe that the average surface temperature of the world has been rising steadily. The average surface temperature can be modelled by $T = 0.02t + 15$ where $T$ is the temperature in Celcius and $t$ is years since 1950.

(a) What do the slope and $T$-intercept represent?

(b) Use the equation to predict the average global temperature in 2050.

$T = 0.02t + 15$

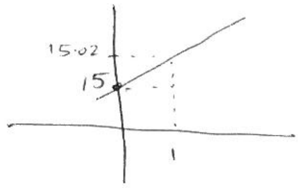
Temp ↑ (T)  time ↑ (t)

$t=0: \quad T = 15$

$t=1: \quad T = 15.02$

$t=2: \quad T = 15.04$

$\vdots$




a) Slope $= 0.02$ means the Temp is increasing by $0.02°C$ per year

T intercept $= 15$, at the start (in 1950) the average surface temp was $15°C$.

b) Want T in 2050.

$T = 0.02t + 15$ and $t$ = time since 1950

$= 100$ yrs (to 2050)

$\therefore$ Want T when $t = 100$ : $T = 0.02(100) + 15$

$= 17°C$

15. A manufacturing company to produces toasters. The production cost is \$3000 plus \$6 per toaster.

(a) Find a formula for the total cost in terms of the number of toasters.

(b) Sketch the graph of this equation.

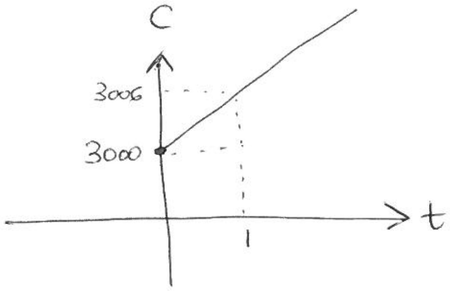
(c) Explain the significance of the slope and y-intercept.

(d) How many toasters would need to be produced so that the total cost is \$6000.

a) Let $C$ = cost, $t$ = nº of toasters.

$\therefore$ $C = 3000 + 6t$

b)




Linear : y int = 3000
slope = 6

This graph only exists for positive $t$.

c) Slope = 6 ie: The cost increases by \$6 for each toaster produced.

y-int = 3000 ie: when $t=0$, $C=3000$
$\therefore$ The initial manufacturing cost is \$3000 before production starts.

d) Want nº of toasters so $C = \$6000$

ie: want $t$ when $C = 6000$

ie: $6000 = 3000 + 6t$

$3000 = 6t$

$500 = t$

$\therefore$ 500 toasters need to be produced.