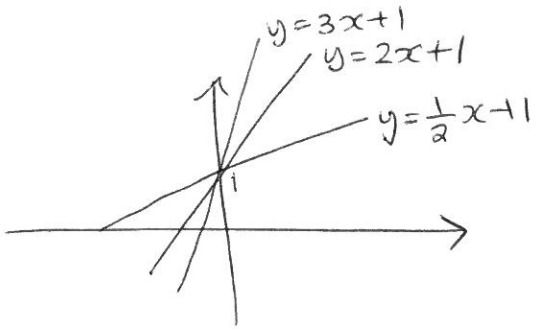
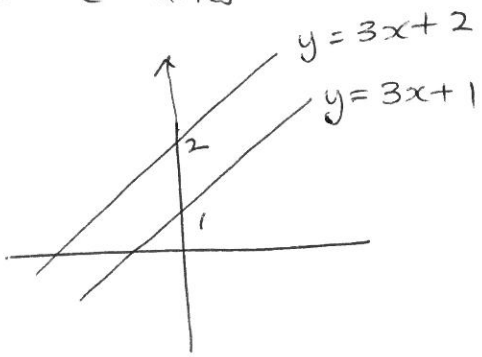
Last time :

We need to be able to picture lines

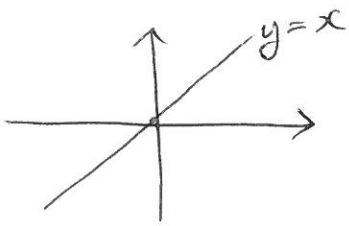




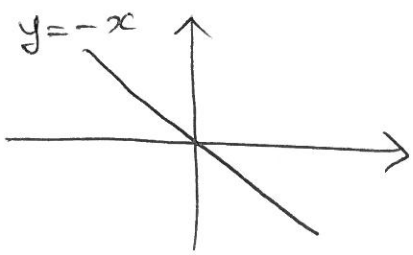
slope = gradient
→ affects steepness

y intercepts
⇒ places it on the plane

$y = mx + b$
↑ gradient
↑ y intercept



$$\text{Gradient} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



Eg 2) a) Line passing through $(5,3)$ and $(8,12)$
x_1, y_1 x_2, y_2

has slope $m = \frac{12-3}{8-5} = \frac{9}{3} = 3$ - pos slope
- sloping up

[note: if I picked $(5,3)$ + $(8,12)$
x_2, y_2 x_1, y_1

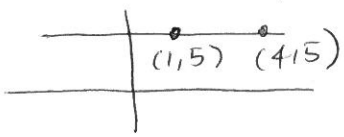
$m = \frac{3-12}{5-8} = \frac{-9}{-3} = 3$.]

b) Line passing through $(3,-4)$ and $(-5,6)$
x_1, y_1 x_2, y_2

$m = \frac{6-(-4)}{-5-3} = \frac{10}{-8} = -\frac{5}{4}$

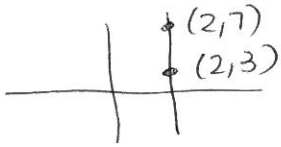
neg slope ∴ Line sloping down

c) Line through $(1,5)$ and $(4,5)$

$m = \frac{5-5}{4-1} = \frac{0}{3} = 0$ Notice 

Horizontal line has 0 slope
Eqn of line is $y=5$

d) Line through $(2,3)$ and $(2,7)$

$m = \frac{7-3}{2-2} = \frac{4}{0}$ ← undefined 

Vertical line has undefined slope.
Eqn of line is $x=2$

Equation of a Line

$y = mx + b$ is called the slope-intercept eqn of the line

where m = gradient
b = y-intercept
and x, y = variables

eg: 3a) Line through $(5,3)$ and $(8,12)$

Let $y = mx + b$

we found $m = 3$.

$\therefore y = 3x + b$

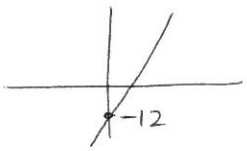
we know $(5,3)$ satisfies the eqn of the line.

$\therefore 3 = 3(5) + b$

$3 = 15 + b$

$b = -12$

$\therefore y = 3x - 12$



b) Line through $(3,-4)$ and $(-5,6)$

Let $y = mx + b$

we found $m = -\frac{5}{4}$ so $y = -\frac{5}{4}x + b$

Since $(-5,6)$ satisfies the eqn of the line.

$$6 = -\frac{5}{4}(5) + b$$

$$6 = \frac{25}{4} + b$$

$$b = 6 - \frac{25}{4} = \frac{24-25}{4} = -\frac{1}{4}$$

$$\therefore y = -\frac{5}{4}x - \frac{1}{4}$$

c) $y = 5$ ← Special cases

d) $x = 2$ ←

The general equation of a line tells us how to recognise lines.

General eqn of a line is $ax + dy + c = 0$ ← Linear Equation

x, y = variables

a, d, c = constants = numbers.

eg 1) $y = 2x - 1$ is a line.

ie! $2x - y - 1 = 0$ so $a = 2$, $d = -1$, $c = -1$

2) $3x - y = 0$ is a line

$a = 3$, $b = -1$, $c = 0$

3) $x = 2$ is a line (vertical line)

ie: $x + 0y - 2 = 0$ $a = 1$, $b = 0$, $c = -2$

4) $x^2 + y + 1 = 0$ is NOT a line

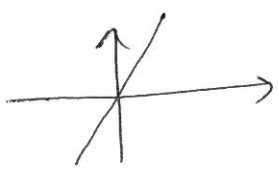
5) $y = \sqrt{x} + 1$ is NOT a line

6) $y = x^{-1} + 2$ is NOT a line

- we like to write our lines in slope-intercept form

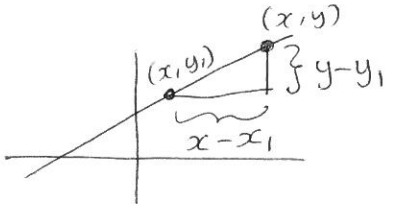
$3x - y = 0$ is same as $y = 3x$

$\therefore m = 3$ + $b = 0$



Point - Slope Eqn of a Line

= another eqn of line thats useful when we know the slope and at least 1 point on line



Let m = slope

(x_1, y_1) = point on line

Let (x, y) = any other point on line

So $$\frac{y - y_1}{x - x_1} = m$$

ie: $$\boxed{y - y_1 = m(x - x_1)}$$

Eg 4a) Find the equation of the line passing through $(2, -1)$ with slope $= 3$.

Line: $y - y_1 = m(x - x_1)$ Here $m = 3$

$(x_1, y_1) = (2, -1)$

$\therefore y - (-1) = 3(x - 2)$

ie: $y + 1 = 3x - 6$

$y = 3x - 7$

b) Line passing through $(6, -2)$ and $(9, 4)$

Line: $y - y_1 = m(x - x_1)$

Let $(x_1, y_1) = (6, -2)$

Finding $m = \frac{4 - (-2)}{9 - 6} = \frac{6}{3} = 2$

$\therefore y - (-2) = 2(x - 6)$

$y + 2 = 2x - 12$

$y = 2x - 14$

Equations of Lines

- general eqn → how to recognise a line
- $y = mx + b$ = slope-intercept eqn → useful to picture lines
- $y - y_1 = m(x - x_1)$ = point-slope eqn → useful to work with

Graphing Lines

eg 5) Sketch the graph of the equation $2x - 3y - 12 = 0$.

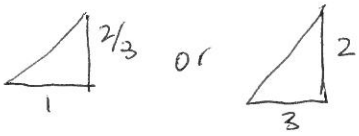
note: recognise this is a line

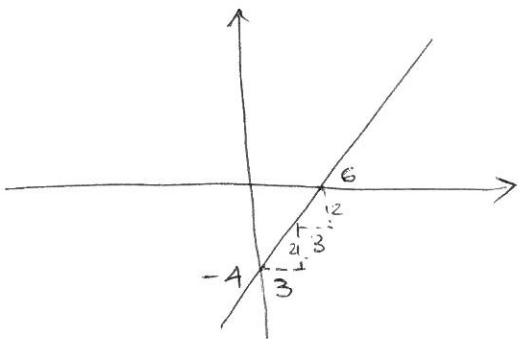
method ①: write in slope-intercept form

$$3y = 2x - 12$$
$$y = \frac{2}{3}x - \frac{12}{3}$$
$$y = \frac{2}{3}x - 4$$

So $m = \frac{2}{3}$, y-int $= -4$

pos ⇒ sloping up → for every 1 unit across we go $\frac{2}{3}$ units up

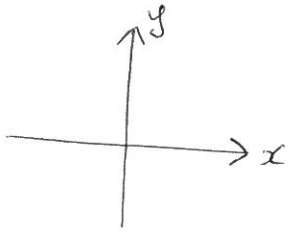




ie: for every 3 units across we go 2 units up

Method ②: Find intercepts.

y-intercept = where graph cuts y axis
= where x-value is 0



x-intercept = where graph cuts x-axis
= where y-value is 0

So $2x - 3y - 12 = 0$.

y int: Let $x = 0$: $0 - 3y - 12 = 0$

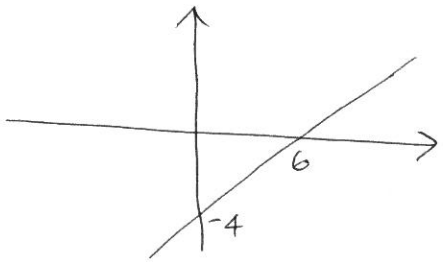
$-3y = 12$

$y = -4$

x int: Let $y = 0$: $2x - 0 - 12 = 0$

$2x = 12$

$x = 6$



We only need 2 points to draw a line.

Finding Intercepts: x-intercept → Let $y = 0$
y-intercept → Let $x = 0$