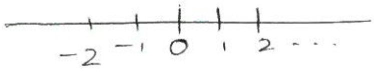
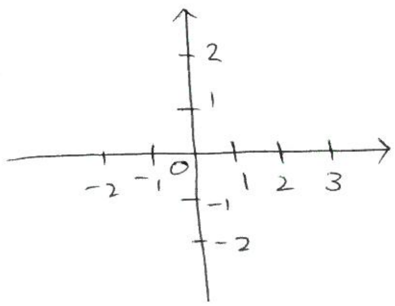
Cartesian Plane

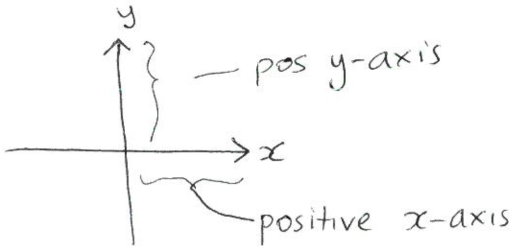
- Number Line 

← 1 dim

- Cartesian Plane 

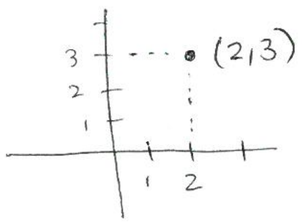
← 2 dim

- lines called axes: horizontal line = x-axis
  vertical line = y-axis



Points

- describe location using points.



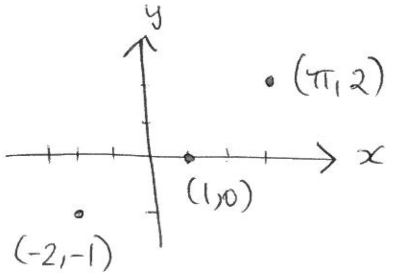
$(2, 3)$ = position in cartesian plane where $x=2$ + $y=3$.
↑ x value ↑ y value

- point = pair (x, y) of real numbers.
  ↑ x coordinate ↖ y coordinate.

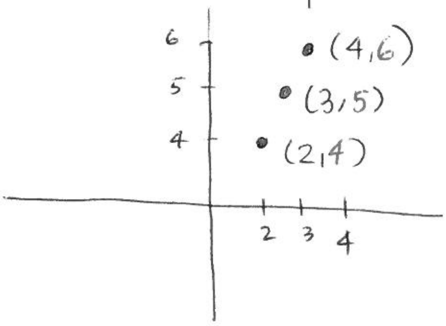
point = coordinate
= ordered pair

- centre $(0,0)$ = origin
- All points on x-axis are of the form $(x,0)$
- " " " y-axis " " " " $(0,y)$

- we should be able to picture points on plane



- Consider certain points



← notice: y-value is 2 more than x-value.

$\therefore$ Here $y = x + 2$

↗ This relationship between $x+y$ describes certain points.

- Think about all points in this relationship $y = x + 2$
- Looks like they lie in a straight line
  - → How can we tell?
    - Everytime we go along 1 unit we go up 1 unit

- In general - an eqn that tells us how to get a y value from an x-value gives us some sort of shape.

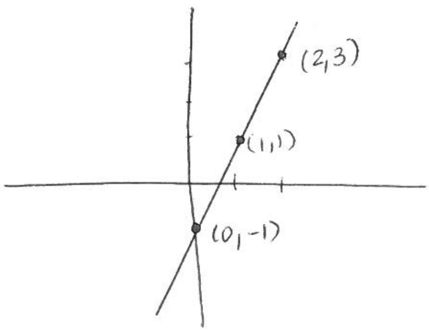
- we're going to concentrate on the eqns that give us lines.

Lines

eg: $y = 2x - 1$

Plot points:

x	-1	0	1	2	3
y	-3	-1	1	3	5



eg: when $x = 2$ then $y = 3$
so we have ordered pair $(2,3)$

Looks like a line

- in fact for every unit we go across we go 2 units up

- Eqn $y = 2x - 1$

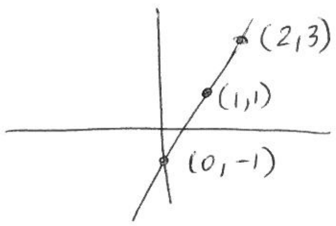
  - we always get straight lines when x has power of 1

  - we write $y = \cdots$ since we think of x as the input and y as the output.

- Line is whole bunch of points joined up
- To draw a line we only need 2 points

How to show points lie on a line

eg) $y = 2x - 1$ + we saw



we know $(0,-1)$, $(2,3)$ lie on this line

Notice what happens when we substitute these into line

sub $(0,-1)$ into $y = 2x - 1$

ie: $-1 = 2(0) - 1$

ie: $-1 = -1$ ✓ which is true.

we say $(0,-1)$ satisfies the eqn of the line

ie: $(0,-1)$ lies on the line

what about $(2,3)$: $3 = 2(2) - 1$

$3 = 3$

$\therefore$ $(2,3)$ lies on this line since it satisfies the eqn of the line

what about $(3,1)$: $1 = 2(3) - 1$

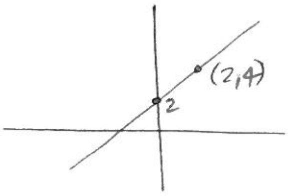
$1 = 5$ ✗ Not true.

$(3,1)$ does not satisfy the eqn of the line

$\therefore$ It does not lie on the line

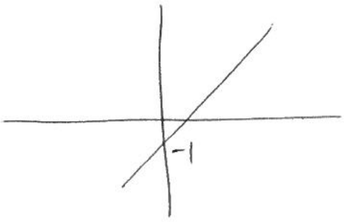
Picturing Lines

Recall: $y = x + 2$



go along 1
+ up 1

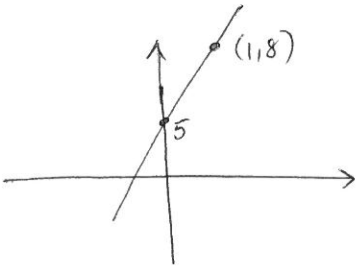
$y = 2x - 1$



go along 1
+ up 2

What about

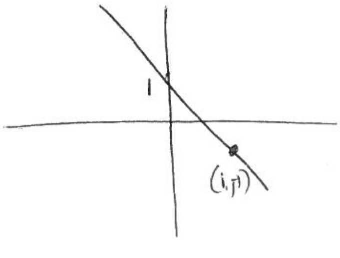
$y = 3x + 5$



for every unit we go across, we go up 3

x	0	1	2
y	5	8	11

$y = -2x + 1$



for every unit we go across we go down by 2

x	0	1	2
y	1	-1	-3

What do you notice?

$y = 2x - 1$

tells us how steep the line is

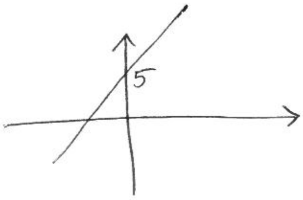
tells us where it cuts the y-axis
ie: y-intercept

We say that a line has the equation $\boxed{y = mx + b}$

where m = gradient/steepness

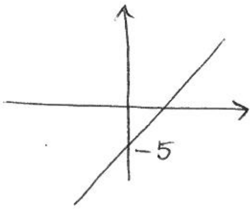
b = y-intercept

Eg1) a) $y = 2x + 5$



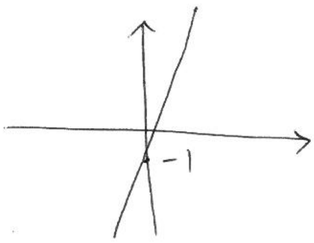
$m = 2$
$b = 5$

b) $y = 2x - 5$



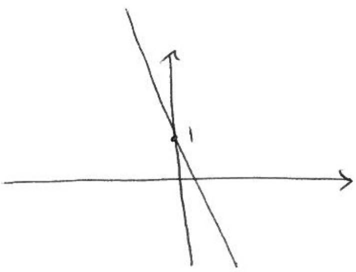
$m = 2$
$b = -5$

c) $y = 4x - 1$



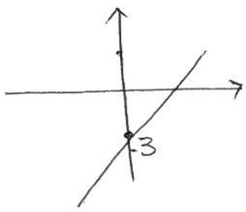
$m = 4$ (steeper)
$b = -1$

d) $y = 1 - 4x$



$m = -4$ (sloping backwards)
$b = 1$

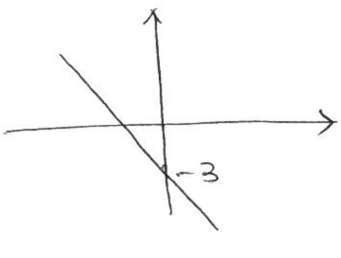
e) $y = x - 3$



$m = 1$ (45° slope)
$b = -3$

* Need to be able to picture these lines in your head when looking at eqn.

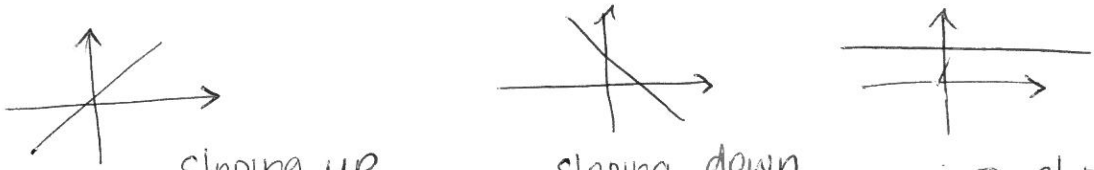
f) $y = -x - 3$



$m = -1$
$b = -3$

Gradient

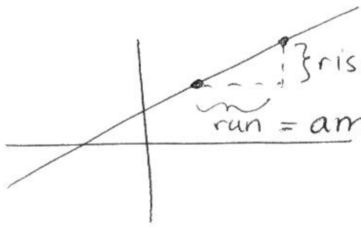
- key to describing lines
- there are lots of different lines



sloping up (pos gradient) | sloping down ($m < 0$) | 0 slope

Gradient = slope of line

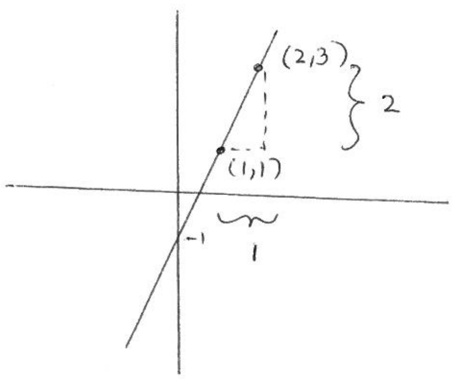
$$= \frac{\text{rise}}{\text{run}}$$



$$= \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

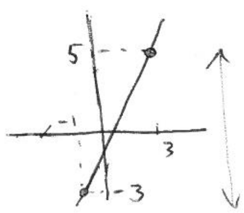
= rate of change

eg: $y = 2x - 1$



$$\frac{\text{rise}}{\text{run}} = \frac{2}{1}$$

- this is true for any two points we pick on this line
- Taking $(-1,-3)$ and $(3,5)$



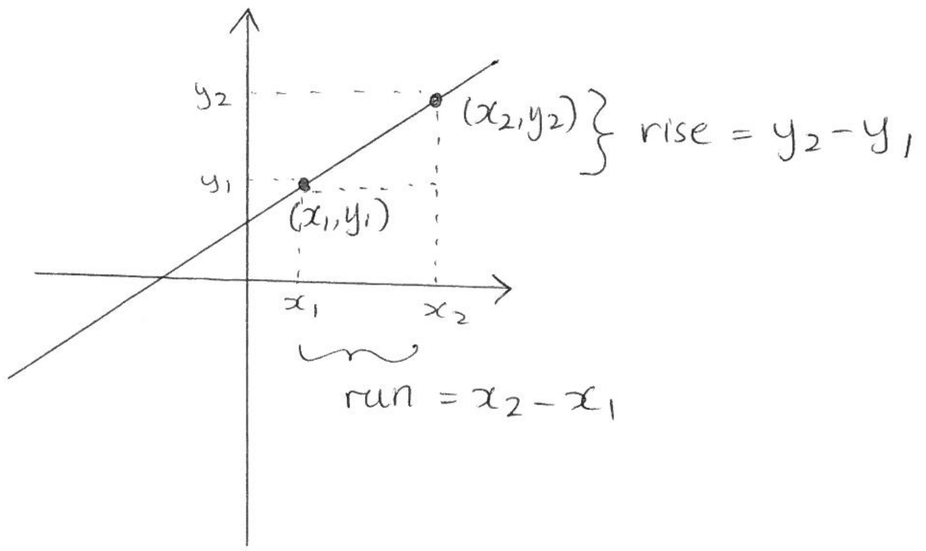
rise = 8

run = 4

$\therefore \frac{\text{rise}}{\text{run}} = 2$

Gradient formula

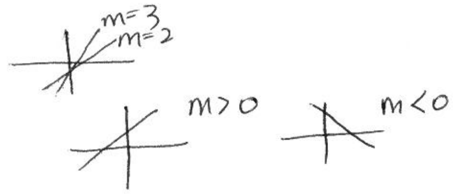
- we can find a formula by noticing
  rise = difference in y-values
  run = difference in x-values



note: use subscripts to distinguish points.
(x_1, y_1) = 1st point
(x_2, y_2) = 2nd point.

$$\boxed{\text{gradient} = m = \frac{y_2 - y_1}{x_2 - x_1}}$$

note: - steeper line = larger gradient
- sloping to right → pos gradient
- sloping to left → neg gradient



- using gradient formula - doesn't matter which point you take first, but you must keep same order with x's as with y's.

eg. Line passing through (5, 3) and (8, 12)

$(5, 3)$ $(8, 12)$
$x_1\ y_1$ $x_2\ y_2$

$$m = \frac{12-3}{8-5} = \frac{9}{3} = 3$$

OR $(5, 3)$ $(8, 12)$
$x_2\ y_2$ $x_1\ y_1$

$$m = \frac{3-12}{5-8} = \frac{-9}{-3} = 3$$