Describing numbers

Even number: 2, 4, 6, 8, 10, ...

multiples of 2.

Call them $2k$ for $k = 1, 2, 3, 4 \ldots$ (also $k = 0, -1, -2, \ldots$)

ie: $2k \quad k \in \mathbb{Z}$

Odd number - sits inbetween even numbers.
- leaves a remainder of 1 when divided by 2.
- even $n^{o} \pm 1$.

$\therefore$ we can say odd number: $2k + 1 \quad k \in \mathbb{Z}$

OR $2k - 1 \quad k \in \mathbb{Z}$

Numbers divisible by 3 $= 3k \quad k \in \mathbb{Z}$

Eg 7 :

7. All odd numbers can be written in the form $2k - 1$, for some integer k. Prove that the product of two odd numbers must be an odd number.

Even number : $2k \quad k \in \mathbb{Z}$

Odd number : $2k-1 \quad k \in \mathbb{Z}$

or $2k+1$

Here we have 2 odd numbers $2k-1$, $2m-1$

Product is $(2k-1)(2m-1)$

[Aim : Show answer is an odd n°
ie : 2x integer ± 1]

$\therefore (2k-1)(2m-1) = 4km - 2k - 2m + 1$

$= 2(2km - k - m) + 1$

Integer since the prod + sum of integers is an integer.

$\therefore$ Product is of the form $2 \times$ Integer $+ 1$

$\therefore$ Product is an odd number.

Solving

- note the difference between factorising + solving
- Remember solving → Have an equation
  → Aim : Find x.

eg 1) Solve $x^2+5x+4=0$.

First factorise : $\left.\begin{array}{l} P=4 \\ S=5 \end{array}\right\} 1, 4$

$\therefore x^2+5x+4 = (x+1)(x+4) = 0$.

2 things mult to give 0

$\therefore x+1=0 \quad , \quad x+4=0$

$x=-1 \quad\quad x=-4$.

2 solutions

[Check ! $x=-1$: $(-1)^2+5(-1)+4=0$ ✓
$x=-4$: $(-4)^2+5(-4)+4=0$ ✓]

2) Solve $9x^3 = 4x$

$9x^3-4x=0$ ← Be careful : Don't divide by x or we lose a soln.

$x(9x^2-4)=0$

$x(3x+2)(3x-2)=0$

$x=0 \, , \frac{2}{3}, -\frac{2}{3}$

3) Solve $x^3 - 8x^2 + 16x = 0$

$$x(x^2 - 8x + 16) = 0$$

$$x(x-4)^2 = 0$$

$$x = 0, 4$$

4) Solve $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$

Perf sq: $\left(x + \frac{1}{3}\right)^2 = 0$

$$\therefore x = -1/3$$

OR $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$

Same as solving: $9x^2 + 6x + 1 = 0$

ie: $(3x)^2 + 2(3x)\cdot 1 + 1^2 = 0$

$$(3x+1)^2 = 0$$

$$\therefore 3x + 1 = 0$$

$$x = -1/3$$

So the eqns $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$ and $9x^2 + 6x + 1 = 0$ are equivalent $\Rightarrow$ They both have same solution set.

Solving using Quadratic Formula

- Some quadratics can't be factorised or don't have nice whole number solutions.
- The quadratic formula gives us solutions.

Remember : Quadratic is of form ax^2+bx+c

Solving $ax^2+bx+c=0$
use quad formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

eg 1) Solve $x^2+5x+3=0$

$(a=1, b=5, c=3)$

$$x = \frac{-5 \pm \sqrt{25-4(3)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25-12}}{2}$$

$$= \frac{-5 \pm \sqrt{13}}{2}$$

Two solutions $x = \frac{-5-\sqrt{13}}{2}$, $x = \frac{-5+\sqrt{13}}{2}$

note: Quad formula gives solutions

2) Solve $x^2 - 2x + 1 = 0$

$(a=1,\ b=-2,\ c=1)$

$$x = \frac{-(-2) \pm \sqrt{4-4}}{2}$$

$$= \frac{2 \pm 0}{2}$$

$$= 1$$

one solution. (same if we used perf sq rule to solve)

3) Solve $2x^2 - x + 1 = 0$

$(a=2,\ b=-1,\ c=1)$

$$x = \frac{1 \pm \sqrt{1-4(2)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{-7}}{4} \leftarrow \text{Problem!}$$

No solution

Notice: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} \leftarrow$ whats inside sq root determines how many solutions well have.

If $b^2 - 4ac > 0 \rightarrow$ 2 solns

$b^2 - 4ac = 0 \rightarrow$ 1 soln

$b^2 - 4ac < 0 \rightarrow$ No solns

$b^2 - 4ac$ = Discriminant.

eg) Solve $2x^2 - 5x + 1 = 0$

Notice discriminant $= b^2 - 4ac$

$= (-5)^2 - 4(2)(1)$

$= 25 - 8$

$= 17$

$\therefore$ We expect 2 solutions.

$$x = \frac{-(-5) \pm \sqrt{17}}{2(2)}$$

$$= \frac{5 \pm \sqrt{17}}{4}$$

$\therefore x = \dfrac{5 + \sqrt{17}}{4}$ $\quad x = \dfrac{5 - \sqrt{17}}{4}$