More Examples : Factorise:

1) $x^3 - x = x(x^2-1) = x(x+1)(x-1)$

2) $a^4 - b^4 = (a^2+b^2)(a^2-b^2)$

$= (a^2+b^2)(a+b)(a-b)$

3) $2x^4 + 16x = 2x(x^3+8)$

$= 2x(x+2)(x^2-2x+4)$

4) $x^6 - y^6 = (x^3+y^3)(x^3-y^3)$

$= (x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

$= (x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$

5) $w^5z^2 + z^5w^2 = w^2z^2(w^3+z^3)$

$= w^2z^2(w+z)(w^2-wz+z^2)$

Laws on Squares + Cubes

$a^2 - b^2 = (a+b)(a-b)$

$a^2 + 2ab + b^2 = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)^2$

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

8. I invest \$1000 in a term deposit. Each year it increases its value by 5.2%.

(a) Find a formula to express the total amount I will have in my account after n years.

(b) How much money will I have after 10 years?

a) 1yr : $1000 + 0.052(1000) = 1000(1.052)$

2yrs : $1000(1.052) + 0.052[1000(1.052)]$

$= 1000(1.052)(1+0.052)$

$= 1000(1.052)(1.052)$

$= 1000(1.052)^2$

3yrs : $1000(1.052)^2 + 0.052(\cancel{\$}1000(0.052)^2)$

$= 1000(1.052)^2(1+0.052)$

$= 1000(1.052)^3$

$\vdots$

* Look for the pattern.

$\therefore$ n yrs : $1000(1.052)^n$

$\therefore$ Amount $= 1000(1.052)^n$

b) After 10 yrs, Amount $= 1000(1.052)^{10}$

$= \$1660.19$

Factorising Quadratics

Quadratic = algebraic expression where the highest power that occurs is 2.

eg: x^2+x+1
$-x^2+5$
$5x^2+4x-3$
x^2

These are quadratics.

$x+3$
$\sqrt{x}+7x$

these are NOT

General form of a quadratic is ax^2+bx+c

- a,b,c = coefficients, constants
  $a \neq 0$
- x = variable

- We want to factorise expressions of this form.
- We know how to deal with certan ones

eg: $x^2-36 = (x+6)(x-6)$ ← Diff of 2 sq

$x^2+16x+64 = (x+8)^2$ ← Perf sq

But what about for any a,b,c ?

Product - Sum method

- lets work backwards.

$$(x+2)(x+3) = x(x+3) + 2(x+3)$$
$$= x^2 + 3x + 2x + 6$$
$$= x^2 + \underline{5}x + \underline{6}$$

notice
$5 = 2+3$

$6 = 2 \times 3$

$\therefore$ Given x^2+5x+6 we want 2 numbers whose

$$\left.\begin{array}{l}\text{product} = 6 \\ \text{sum} = 5\end{array}\right\} 2, 3$$

$\therefore$ Write $x^2 + 5x + 6 = x^2 + 3x + 2x + 6$

$$= x(x+3) + 2(x+3)$$
$$= (x+3)(x+2)$$

eg 2) $x^2 + 9x + 20$

$$\therefore \left.\begin{array}{l}\text{Prod} = 20 \\ \text{sum} = 9\end{array}\right\} 5, 4.$$

so $x^2 + 9x + 20 = x^2 + 5x + 4x + 20$

$$= x(x+5) + 4(x+5)$$
$$= (x+5)(x+4)$$

Do we have to go through whole process?

3) $x^2 + 3x - 10$

$$\left.\begin{array}{l} \text{Prod} = -10 \\ \text{sum} = 3 \end{array}\right\} 5, -2$$

$\therefore x^2 + 3x - 10 = (x+5)(x-2)$

4) $x^2 - 3x - 10$

$$\left.\begin{array}{l} \text{Prod} = -10 \\ \text{Sum} = -3 \end{array}\right\} -5, 2$$

$\therefore x^2 - 3x - 10 = (x-5)(x+2)$

The actual rule:

To factorise a quadratic $ax^2 + bx + c$ find 2 numbers whose Product $= a \times c$, Sum $= b$

eg) Factorise $2x^2 + 9x + 4$.

$\therefore$ want 2 numbers whose $\left.\begin{array}{l} \text{prod} = 2 \times 4 = 8 \\ \text{sum} = 9 \end{array}\right\} 1, 8$

This time $2x^2 + 9x + 4 \neq (x+1)(x+8)$

Instead $2x^2 + 9x + 4 = 2x^2 + x + 8x + 4$

$= x(2x+1) + 4(2x+1)$

$= (2x+1)(x+4)$

2) $3x^2 + x - 10$

$$\left.\begin{aligned} \text{Prod} &= -10 \times 3 = -30 \\ \text{Sum} &= 1 \end{aligned}\right\} -5, 6$$

$\therefore 3x^2 + 6x - 5x - 10$

$= 3x(x+2) - 5(x+2)$

$= (x+2)(3x-5)$

3) $6x^2 - x - 12$

$$\left.\begin{aligned} \text{Prod} &= -12 \times 6 = -72 \\ \text{sum} &= -1 \end{aligned}\right\} -9, 8$$

$\therefore 6x^2 - 9x + 8x - 12$

$= 3x(2x-3) + 4(2x-3)$

$= (2x-3)(3x+4)$

(* Try Q5 - Problem set)

More Algebraic Fractions

Simplify:

1) $$\frac{x^2+6x+9}{x+3} = \frac{(x+3)^2}{x+3} = x+3$$ (← Note: we're assuming $x \neq -3$)

2) $$\frac{x+y}{x^2-4y^2} \times \frac{6y-3x}{2x+2y}$$

$$= \frac{x+y}{(x+2y)(x-2y)} \cdot \frac{3(2y-x)}{2(x+y)}$$

$$= \frac{\cancel{x+y}}{(x+2y)\cancel{(x-2y)}} \cdot \frac{-3\cancel{(x-2y)}}{2\cancel{(x+y)}}$$

$$= \frac{-3}{2(x+2y)}$$

3) $$\frac{x^2-y^2}{\frac{1}{x}+\frac{1}{y}} = x^2-y^2 \div \left(\frac{1}{x}+\frac{1}{y}\right)$$

$$= x^2-y^2 \div \left(\frac{y+x}{xy}\right)$$

$$= x^2-y^2 \times \frac{xy}{y+x}$$

$$= \cancel{(x+y)}(x-y) \cdot \frac{xy}{\cancel{y+x}}$$

$$= xy(x-y)$$

4) $$\frac{4x^2-1}{2x^2-9x+4} \div \frac{6x+3}{4-x}$$

$$= \frac{(2x+1)(2x-1)}{(2x-1)(x-4)} \div \frac{3(2x+1)}{4-x}$$

$P = 8$
$S = -9$ $\Big\}\ -1 ; -8$

$2x^2 - x - 8x + 4$
$= x(2x-1) - 4(2x-1)$
$= (2x-1)(x-4$

$$= \frac{\cancel{(2x+1)}\cancel{(2x-1)}}{\cancel{(2x-1)}(x-4)} \times \frac{4-x}{3\cancel{(2x+1)}}$$

$$= \frac{4-x}{3(x-4)}$$

$$= \frac{-(x-4)}{3(x-4)}$$

$$= -\frac{1}{3}$$

5) $$\frac{2}{x^2-1} - \frac{1}{x^2-x} + \frac{5}{x^2+x}$$

$$= \frac{2}{(x+1)(x-1)} - \frac{1}{x(x-1)} + \frac{5}{x(x+1)}$$

$$= \frac{2x - (x+1) + 5(x-1)}{x(x+1)(x-1)}$$

$$= \frac{2x - x - 1 + 5x - 5}{x(x+1)(x-1)}$$

$$= \frac{6x-6}{x(x+1)(x-1)} = \frac{6(x-1)}{x(x+1)(x-1)} = \frac{6}{x(x+1)}$$

6) $$\frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{x^2-y^2}$$

$$= \frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{(x+y)(x-y)}$$

$$= \frac{x(x+y) - y(x-y) - 2xy}{(x+y)(x-y)}$$

$$= \frac{x^2 + xy - xy + y^2 - 2xy}{(x+y)(x-y)}$$

$$= \frac{x^2 - 2xy + y^2}{(x+y)(x-y)}$$

$$= \frac{(x-y)^{\cancel{2}}}{(x+y)\cancel{(x-y)}}$$

$$= \frac{x-y}{x+y}$$