Factorising

① Take out common factors.
- we've already looked at this method.

eg 1) $3xy - 6x^2y + 12xy^2 = 3xy(1 - 2x + 4y)$

2) $x(x+1) + 5(x+1) = (x+1)(x+5)$

3) $x^2 + x + 5x + 5$

$= x(x+1) + 5(x+1)$

$= (x+1)(x+5)$

← Look out for common factors in pairs

4) $x^2 + 3x - 2xy - 6y = x(x+3) - 2y(x+3)$

$= (x+3)(x-2y)$

5) $m^3 + m^2 + m + 1 = m^2(m+1) + 1(m+1)$

$= (m^2+1)(m+1)$

6) $s^2 + st + 3s^3 + 3s^2t = s[s + t + 3s^2 + 3st]$

$= s[s+t + 3s(s+t)]$

$= s(s+t)(1+3s)$

② Difference of 2 Squares

Notice: $(a+b)(a-b) = a^2 + ab - ab - b^2$

$= a^2 - b^2$

So $\boxed{a^2 - b^2 = (a+b)(a-b)}$

eg 1) $x^2 - 4 = x^2 - 2^2$

$= (x+2)(x-2)$

check $(x+2)(x-2)$
$= x^2 + \cancel{2x} - \cancel{2x} - 4$ ✓

2) $x^2 - 25 = x^2 - 5^2$

$= (x+5)(x-5)$

3) $9x^2 - 4 = (3x)^2 - 2^2$

$= (3x-2)(3x+2)$

4) $36x^2 - 1 = (6x+1)(6x-1)$

5) $x^3 - 16x = x(x^2 - 16)$

$= x(x+4)(x-4)$

Note:

- This rule is also useful when expanding

eg 1) Expand $(x-\sqrt{2})(x+\sqrt{2}) = x^2 - 2$

2) Expand $(x^2+9)(x-3)(x+3)$

$$= (x^2+9)(x^2-9)$$

$$= (x^2)^2 - 9^2$$

$$= x^4 - 81$$

- It's also useful for rationalising the denominator

eg 1) $$\frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{3-\sqrt{2}}{3^2-2}$$

$$= \frac{3-\sqrt{2}}{7}$$

2) $$\frac{4}{1-\sqrt{7}} \times \frac{1+\sqrt{7}}{1+\sqrt{7}}$$

$$= \frac{4(1+\sqrt{7})}{1-7}$$

$$= \frac{4+4\sqrt{7}}{-6} = \frac{\cancel{2}(\ \ +2\sqrt{7})}{-\cancel{6}3} = \frac{2+2\sqrt{7}}{-3} = -\frac{2}{3} - \frac{2\sqrt{7}}{3}$$

(* Try Q1 - Problem set)

③ Perfect Square Rules

Notice:

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+ab+ab+b^2\\ &= a^2+2ab+b^2\end{aligned}$$

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a^2-ab-ab+b^2\\ &= a^2-2ab+b^2\end{aligned}$$

so

$$\boxed{\begin{aligned}(a+b)^2 &= a^2+2ab+b^2\\ (a-b)^2 &= a^2-2ab+b^2\end{aligned}}$$

expanding

factorising

So we have a quick way of expanding.

eg:
$$\begin{aligned}(x+3)^2 &= x^2+2(3)x+3^2\\ &= x^2+6x+9.\end{aligned}$$

But we also want to recognise the pattern + factorise.

$$\begin{aligned}x^2+6x+9 &= x^2+2(3)x+(3)^2\\ &= (x+3)^2\end{aligned}$$

eg 2) $x^2 - 14x + 49$

$= x^2 - 2(7)x + (7)^2$ $\quad (b = 7)$

$= (x-7)^2$

3) $x^2 + 10x + 25$

$= x^2 + 2(5)x + (5)^2$ $\quad (b = 5)$

$= (x+5)^2$

4) $$\begin{array}{ccc} 4x^2 & + \; 20x & + \; 25 \\ \uparrow & \uparrow & \uparrow \\ (2x)^2 & 2(2x)(5) & 5^2 \end{array}$$

$= (2x+5)^2$

5) $$\begin{array}{ccc} x^2 & - \; 6xy & + \; 9y^2 \\ & \uparrow & \uparrow \\ & 2 \cdot x \cdot (3y) & (3y)^2 \end{array}$$

$= (x-3y)^2$

Expanding also becomes quicker.

Expand

eg) $(x-10)^2 = x^2 - 20x + 100$

2) $(5x+3)^2 = 25x^2 + 30x + 9$

3) $(x+2y)^2(x-2y)^2 = \left((x+2y)(x-2y)\right)^2$ ← diff of 2 sq

$= (x^2 - 4y^2)^2$

$= (x^2)^2 - 2(x^2)(4y^2) + (4y^2)^2$

$= x^4 - 8x^2y^2 + 16y^4$

4) $(7x-2)^2 - (4x+5)^2$

$= 49x^2 - 28x + 4 - (16x^2 + 40x + 25)$

$= 49x^2 - 28x + 4 - 16x^2 - 40x - 25$

$= 33x^2 - 68x - 21$

(* Try Q2 - Problem Set)

Laws on Cubes

$$\boxed{\begin{aligned} a^3 - b^3 &= (a-b)(a^2+ab+b^2) \\ a^3 + b^3 &= (a+b)(a^2-ab+b^2 \end{aligned}}$$

Proof:

$$(a-b)(a^2+ab+b^2) = a^3 + \cancel{a^2b} + \cancel{ab^2} - \cancel{a^2b} - \cancel{ab^2} - b^3$$
$$= a^3 - b^3$$

$$(a+b)(a^2-ab+b^2) = a^3 + \cancel{a^2b} + \cancel{ab^2} + \cancel{ba^2} - \cancel{ab^2} + b^3$$
$$= a^3 + b^3$$

eg 1) Factorise $x^3 - 8 = x^3 - 2^3$

$$= (x-2)(x^2+2x+4)$$

2) Factorise $x^3 + 64 = x^3 + 4^3$

$$= (x+4)(x^2-4x+16)$$

3) Factorise $27x^3 - y^3 = (3x)^3 - y^3$

$$= (3x-y)(9x^2+3xy+y^2)$$