Simplifying Algebraic Fractions

eg: $\frac{x}{2x} = \frac{x \times 1}{x \times 2} = \frac{1}{2}$

* can cancel common factors from top + bottom

eg 1) $\frac{3x}{15x^2} = \frac{1}{5x}$

2) $\frac{2xy^2}{6x^2y} = \frac{y}{3x}$

3) $\frac{x+1}{x}$ ← Here we can't cancel
Dont cancel x

4) $\frac{3x+3}{x+1}$ ← cant cancel until we factorise.

$= \frac{3(x+1)}{(x+1)}$

$= 3$

* Cancel Factors, not terms

* ∴ Factorise top + bottom first.

$\frac{3x+3}{x+1}$ ← WRONG!

eg1) $\frac{x^2-2x}{x-2} = \frac{x(x-2)}{x-2} = x$

2) $\frac{x^2-2x}{x-2} \times \frac{3}{x} = \frac{x(x-2)}{x-2} \times \frac{3}{x} = 3$

3) $\frac{x^2+6x}{x} \times \frac{x^2}{x+6} = \frac{x(x+6)}{x} \times \frac{x^2}{x+6} = x^2$

4) $\frac{5xy-15y}{6y^2} \times \frac{x+y}{4x-12} = \frac{5y(x-3)}{6y^2} \times \frac{x+y}{4(x-3)}$

$= \frac{5(x+y)}{24y}$

5) $\frac{x^2-5x}{2x+10} \times \frac{3x+15}{4x} = \frac{x(x-5)}{2(x+5)} \times \frac{3(x+5)}{4x}$

$= \frac{3(x-5)}{8}$

6) $\frac{x^2+2x}{5} \div \frac{2x+4}{20} = \frac{x^2+2x}{5} \times \frac{20}{2x+4}$

$= \frac{x(x+2)}{5} \times \frac{20}{2(x+2)}$

$= 2x$

7) $\dfrac{\frac{8x+24}{9}}{\frac{4x+12}{12x+12}}$

$= \dfrac{8x+24}{9} \div \dfrac{4x+12}{12x+12}$

$= \dfrac{8x+24}{9} \times \dfrac{12x+12}{4x+12}$

$= \dfrac{8(x+3)}{9_3} \times \dfrac{12^4(x+1)}{4(x+3)}$

$= \dfrac{8(x+1)}{3}$

when mult + div algebraic fractions.

- → factorise first
- → cancel factors, not terms
- → cancel factors from top + bottom

Adding + Subtracting Algebraic Fractions

Simplify (or write as a single fraction)

eg 1) $\frac{x+1}{3} + \frac{y}{2} = \frac{2(x+1)}{6} + \frac{3y}{6}$ * common Denominator

$= \frac{2x+2+3y}{6}$

2) $\frac{x+1}{3y} - \frac{4}{3x} = \frac{x(x+1)}{3xy} - \frac{y(4)}{3xy}$

$= \frac{x^2+x-4y}{3xy}$ $\left(\text{or } \frac{x^2-4y+x}{3xy}\right)$

3) $\frac{4}{3x} - \frac{x}{x+2} = \frac{4(x+2)}{3x(x+2)} - \frac{3x(x)}{3x(x+2)}$

$= \frac{4(x+2)-3x^2}{3x(x+2)}$

$= \frac{4x+8-3x^2}{3x(x+2)}$

4) $\frac{4}{x^2+2x} - \frac{x}{x+2} = \frac{4}{x(x+2)} - \frac{x}{x+2}$

$= \frac{4}{x(x+2)} - \frac{x^2}{x(x+2)}$

$= \frac{4-x^2}{x(x+2)}$

5) $\frac{p}{p-q} + \frac{q}{q-p} = \frac{p}{p-q} + \frac{q}{-(p-q)}$

$= \frac{p}{p-q} - \frac{q}{p-q}$

$= \frac{p-q}{p-q}$

$= 1$

6) $\frac{1+x^{-1}}{1-x^{-1}} = \left(1+\frac{1}{x}\right) \div \left(1-\frac{1}{x}\right)$

$= \frac{x+1}{x} \div \frac{x-1}{x}$

$= \frac{x+1}{\cancel{x}} \times \frac{\cancel{x}}{x-1}$

$= \frac{x+1}{x-1}$

Solving

eg1) Solve $\frac{x-2}{3} - \frac{4x-1}{5} = 2$

← Equation
· Aim → find x.

$\frac{x-2}{3}^{\times 15} - \frac{4x-1}{5}^{\times 15} = 2^{\times 15}$

$5(x-2) - 3(4x-1) = 30$

$5x - 10 - 12x + 3 = 30$

$-7x - 7 = 30$

$-7x = 37$

$x = -\frac{37}{7}$

2) Solve $\frac{1}{x}^{\times 3x} = \frac{4}{3x}^{\times 3x} + 1^{\times 3x}$

$3 = 4 + 3x$

$-1 = 3x$

$x = -1/3$

3) Solve $\frac{2x-7}{2x+4} = \frac{2}{3}$

$3(2x-7) = 2(2x+4)$

$6x - 21 = 4x + 8$

$2x = 29$

$x = \frac{29}{2}$

4) Solve for x in terms of y:

$$\frac{1+3xy}{4y} = x+1$$

↖ means $x =$ ☁ (ys in there.)

$$1+3xy = 4y(x+1)$$

$$1+3xy = 4xy+4y$$

$$1-xy = 4y$$

$$-xy = 4y-1$$

$$x = \frac{4y-1}{-y} = -\frac{(4y-1)}{y} = -\frac{4y+1}{y} = \frac{1-4y}{y}$$

5) Solve for x in terms of y:

$$\frac{3+x}{4y^2} = \frac{2x+1}{3y}$$

$$\frac{3+x}{4y^2} \times 12y^2 = \frac{2x+1}{3y} \times 12y^2 \qquad (12y^2 \text{ is the LCM})$$

$$3(3+x) = 4y(2x+1)$$

$$9+3x = 8xy+4y \qquad \text{Need } x\text{s together}$$

$$3x-8xy = 4y-9$$

$$x(3-8y) = 4y-9$$

$$x = \frac{4y-9}{3-8y}$$

eg1) Solve $x^2 = 25$

Here $x = \pm 5$

Note: $\sqrt{25} = +5$ ← sq root is pos.

But when solving $x^2 = 25$

$x = \pm\sqrt{25}$

2) Solve $x^2 = 7$

$x = \pm\sqrt{7}$

3) Solve $(x-1)^2 = 49$

$\therefore \quad x - 1 = \pm 7$

ie: $x - 1 = 7$ or $x - 1 = -7$

$x = 8$ $\qquad x = -6$

4) Solve for r if $F = G\frac{m_1 m_2}{r^2}$

Aim: $r =$ ☺

$\therefore Fr^2 = Gm_1m_2$

$r^2 = \frac{Gm_1m_2}{F}$

$\therefore r = \pm\sqrt{\frac{Gm_1m_2}{F}}$

Repeating decimals

- remember we said that repeating decimals can be written as fractions:

eg1) $0.3333\ldots$

let $x = 0.3333\ldots$ — ①

$10x = 3.3333\ldots$ — ②

② − ① : $10x - x = 3$

$9x = 3$

$x = \frac{3}{9} = \frac{1}{3}$

2) $0.272727\ldots = 0.\overline{27}$

let $x = 0.272727\ldots$

$100x = 27.2727\ldots$

$100x - x = 27$

$99x = 27$

$x = \frac{27}{99} = \frac{3}{11}$