Intro to Algebra

- use letters to represent what numbers are doing
- usually x, y, z , a, b, c , m, n , s, t ...
- use it to make a general statement

Arithmetic : $4 + 5 = 5 + 4$

Algebra : $x + y = y + x$

we can see here $x=4, y=5$
But this is true for all numbers (commutativity)
so we express it algebraically.

- expressions involving algebra are called Algebraic Expressions
- convention : Instead of writing

$a \times b = ab$

$a \div b = \frac{a}{b}$

Eg 1) $4a = 4 \times a = a + a + a + a$

2) $a^4 = a \times a \times a \times a$

3) $4 + a = 4 + a$

4) $\frac{a}{a} = 1 \quad (a \neq 0)$

5) $\frac{x}{3} = \frac{1}{3} x$

6) $-\frac{x}{3} = \frac{-x}{3} = \frac{x}{-3} = -\frac{1}{3} \cdot x$

7) $\frac{x}{3/4} = \frac{4x}{3}$

Adding + Subtracting

eg) $a + 2a + b + 3b$

- Terms → separated by + and − signs
- Like terms = terms with same letters.
- can only combine like terms.

$a + 2a + b + 3b = 3a + 4b$

(↑ ↑ like terms; ↑ ↑ like terms)

- The same rules we used with numbers also apply to algebra (eg: order of ops, rearranging).

eg) $2x - 3y + 3x - 5y$

$= (+2x)(-3y)(+3x)(-5y)$

$= 2x + 3x - 3y - 5y$

$= 5x - 8y$ ← we have collected like terms.

2) Simplify $4x^2 + 3y^2 - x^2 = 4x^2 - x^2 + 3y^2 = 3x^2 + 3y^2$

3) Simplify $2x + xy - x = 2x - x + xy = x + xy$

Multiplying

eg 1) $2x \times 3y = 2 \times x \times 3 \times y$

$= 6xy$

Do them in order that suits us

2) $2x \times 3x = 6x^2$

3) $\sqrt{2}x \times \sqrt{2}y = 2xy$

4) $-5x \times \frac{y}{2} = -\frac{5}{2}xy$

5) $-5x \times -3xy = 15x^2y$

(* Try Prob Set Q1)

Dealing with brackets

- Neg out the front → when you remove brackets the sign will change.

$-(2x+1) = -2x-1$

$-(2x-1) = -2x+1$

$-(-2x+1) = 2x-1$

$+(-2x+1) = -2x+1$ ← pos out front, nothing changes.

$= 1-2x$

eg 1) Simplify $(2x-2)-(x+3)$

$= 2x-2-x-3$

$= x-5$

- Numbers out the front → Remember distributive law

$a(b+c) = ab+ac$ (expand →, ← factorise)

eg 1) $3(2x+1) = 6x+3$

2) $-3(2x+1) = -6x-3$

3) $-4(5x-y) = -20x+4y$

4) $3(x+y)-x(x+2) = 3x+3y-x^2-2x$

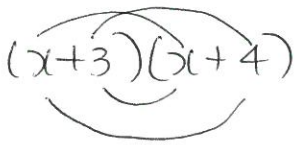
$= x+3y-x^2$

• More than 1 bracket :

eg) Expand $(x+3)(x+4)$ $= x(x+4) + 3(x+4)$

$= x^2 + 4x + 3x + 12$

$= x^2 + 7x + 12$

or 

eg 2) $(x+3)(x-4)$ $= x(x-4) + 3(x-4)$

$= x^2 - 4x + 3x - 12$

$= x^2 - x - 12$

3) $(4x-3)(5x-2)$ $= 4x(5x-2) - 3(5x-2)$

$= 20x^2 - 8x - 15x + 6$

$= 20x^2 - 23x + 6$

4) $(2x+3)^2$ $= (2x+3)(2x+3)$

$= 2x(2x+3) + 3(2x+3)$

$= 4x^2 + 6x + 6x + 9$

$= 4x^2 + 12x + 9$

(This is NOT $4x^2+9$)

(* Try Prob set Q2)

Substitution

= putting numbers where letters are

eg: Speed = $\frac{\text{Dist}}{\text{Time}}$ ie: $S = \frac{D}{T}$

I travelled 5km in 20 mins

$\therefore$ my Speed = $\frac{5\text{km}}{20\text{ mins}} = 0.25\text{ km/min}$

eg) Find the value of $5(x^2+x)$ given $x=4$

$5(x^2+x) = 5(4^2+4) = 5(20) = 100$

eg) The relationship between Farenheit & Celcius is given by the formula $C = \frac{5}{9}(F-32)$

$68^\circ F$ is same as $C = \frac{5}{9}(68-32)$

$= \frac{5}{9}(36)$

$= 20^\circ C$

$100^\circ F$ is same as $C = \frac{5}{9}(100-32) = 37.8^\circ C$

We know 0° is the freezing point of water

The equivalent temp in farenheit is

$0 = \frac{5}{9}(F-32)$ ← Need this to be 0

$\therefore F = 32$

But what about other values of C?

Solving

eg) If $x+4=9$, find x

↖ equation

Aim: find x : $x=$ ☺

$$x+4=9$$
$$\quad -4 \quad -4$$
$$x=9-4$$
$$x=5$$

* what you do to one side of eqns you must do to the other

eg 2) Solve $5-x=2$

$$\quad -5 \quad -5$$
$$-x=2-5$$
$$\quad x-1 \quad x-1$$
$$-x=-3$$
$$x=3$$

* look at the last thing done + undo it