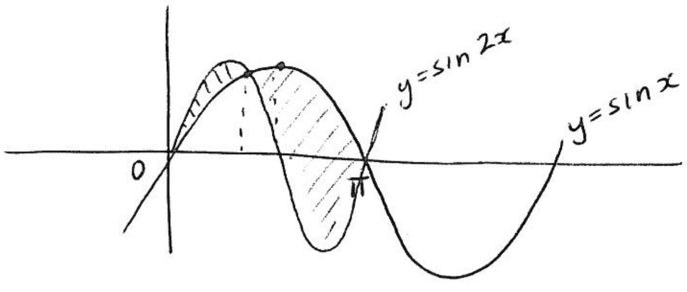
, Find the area enclosed by $y = \sin x$ and $y = \sin 2x$ for $0 \le x \le \pi$.

Draw



Find pt of int:

$$\sin x = \sin 2x$$

$$\sin x = 2\sin x \cos x$$

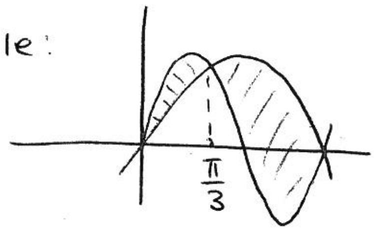
$$2\sin x \cos x - \sin x = 0$$

$$\sin x (2\cos x - 1) = 0$$

$$\sin x = 0 \qquad \cos x = 1/2$$

$$x = 0, \pi, \cdots \qquad x = \pi/3$$

Ie:



$$\text{Area} = \int_0^{\pi/3} \sin 2x - \sin x \, dx + \int_{\pi/3}^{\pi} \sin x - \sin 2x \, dx$$

$$= \left[-\tfrac{1}{2}\cos 2x + \cos x\right]_0^{\pi/3} + \left[-\cos x + \tfrac{1}{2}\cos 2x\right]_{\pi/3}^{\pi}$$

$$= \left[\left(-\tfrac{1}{2}\cos\tfrac{2\pi}{3} + \cos\tfrac{\pi}{3}\right) - \left(-\tfrac{1}{2}\cos 0 + \cos 0\right)\right] + \left[\left(-\cos\pi + \tfrac{1}{2}\cos 2\pi\right) - \left(-\cos\tfrac{\pi}{3} + \tfrac{1}{2}\cos\tfrac{2\pi}{3}\right)\right]$$

$$= \left[\left[-\tfrac{1}{2}\left(-\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)\right] - \left[-\tfrac{1}{2}(1) + 1\right]\right] + \left[\left[1 + \tfrac{1}{2}(1)\right] - \left[-\tfrac{1}{2} + \tfrac{1}{2}\left(-\tfrac{1}{2}\right)\right]\right]$$

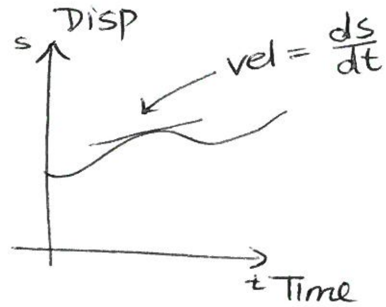
$$= \frac{3}{4} - \frac{1}{2} + \frac{3}{2} - \left(-\frac{3}{4}\right)$$

$$= \frac{10}{4} = 2\frac{1}{2}$$

Applications of Integration

Motion in a straight line

Remember



s = displacement

diff, integ

$\frac{ds}{dt}$ = velocity

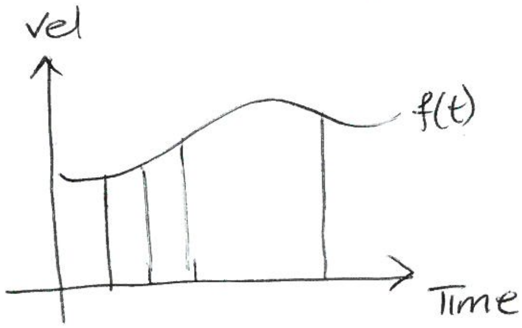
diff, integ

$\frac{d^2s}{dt^2}$ = acceleration

we can also go the other way.

ie: Given velocity we can integrate to find displacement.

But what about areas under curve



← Area = Displacement

ie: Area = sum of rectangles

area of each rect = height × width

$= f(t) \times \Delta t$

= velocity × time

= Displacement.

12. A car starts from rest at $s = 4$ metres from the origin, and has velocity at time t (measured in seconds) given by $v(t) = t^2 - 5t$.

(a) Find the displacement function for the car.

(b) Find the displacement at time $t = 7$ seconds.

a) $s = \int v(t)\ dt$

$\therefore s = \int t^2 - 5t\ dt$

$= \frac{t^3}{3} - 5\frac{t^2}{2} + c$

Need to find c → we know when $t=0$, $s=4$

$\therefore 4 = 0 - 0 + c$

$\therefore c = 4$

$\therefore s(t) = \frac{t^3}{3} - \frac{5t^2}{2} + 4$

b) We want s when $t=7$

ie: $s(7) = \frac{7^3}{3} - \frac{5(7)^2}{2} + 4$

$= \frac{-25}{6}$

$= -4.17\ m$

13. Find the displacement function of a particle moving with velocity $v(t) = \cos \pi t$ along a straight line, when $s(0) = 4$.

$$\text{Disp} = \int v(t)\,dt = \int \cos \pi t \; dt$$

$$= \frac{1}{\pi} \sin \pi t + C$$

Finding C : $s(0) = 4 \rightarrow$ $4 = \frac{1}{\pi} \sin 0 + C$

$\therefore C = 4$

$$\therefore s(t) = \frac{1}{\pi} \sin \pi t + 4$$

14. A particle is moving along a straight line with acceleration given by the function $a(t) = 2t - 4$ where t is the time in seconds. Its initial velocity is $3m/s$.

(a) Find the velocity function at time t.

(b) Find the displacement during the time period $1 \leq t \leq 10$.

(c) Find the distance travelled during the time period $1 \leq t \leq 10$

were told . accel $= a(t) = 2t - 4$

. initial vel $= 3 \Rightarrow$ ie: when $t=0, V=3$.

a) vel $= v(t) = \int a(t)\,dt$

$$= \int 2t - 4 \; dt$$

$$= \frac{2t^2}{2} - 4t + C$$

Finding C : $\Rightarrow t=0 + V=3$: $3 = 0 - 0 + C$

$\therefore C = 3$

$$\therefore v(t) = t^2 - 4t + 3$$

b) Displacement $= s(t) = \int v(t)\,dt$

$$= \int t^2 - 4t + 3 \; dt$$

$$= \frac{t^3}{3} - \frac{4t^2}{2} + 3t + C$$

But we want displacement for $1 \leq t \leq 10$

ie: $s(t) = \int_1^{10} t^2 - 4t + 3 \; dt$

$$= \left[\frac{t^3}{3} - 2t^2 + 3t \right]_1^{10}$$

$$= \left(\frac{10^3}{3} - 2(10)^2 + 3(10) \right) - \left(\frac{1}{3} - 2 + 3 \right)$$

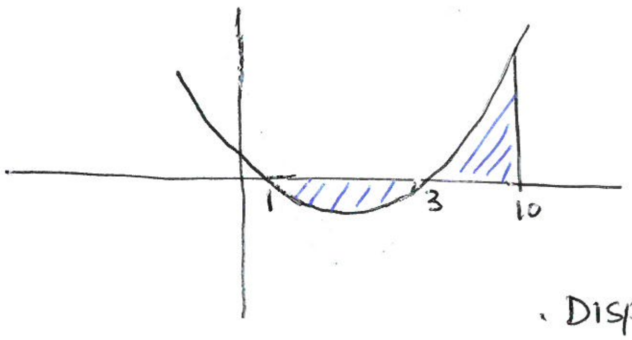
$$= \left(\frac{490}{3} \right) - \left(\frac{4}{3} \right)$$

$$= \frac{486}{3}$$

$$= 162$$

Remember displacement = position from start.

This is different to distance = total amount travelled.



$v(t) = t^2 - 4t + 3$

$= (t-3)(t-1)$

- Displacement → adds areas above + subtracts areas below
- Dist → need to calculate total area.

$$\therefore \text{Dist} = \left| \int_1^3 t^2 - 4t + 3 \ dt \right| + \int_3^{10} t^2 - 4t + 3 \ dt$$

$$= \left| \left[\frac{t^3}{3} - 2t^2 + 3t \right]_1^3 \right| + \left[\frac{t^3}{3} - 2t^2 + 3t \right]_3^{10}$$

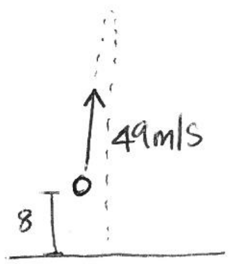
$$= \left| (9 - 18 + 9) - \left(\frac{1}{3} - 2 + 3 \right) \right| + \left(\frac{1000}{3} - 200 + 30 \right) - (9 - 18 + 9)$$

$$= \left| -\frac{4}{3} \right| + \frac{490}{3}$$

$$= \frac{494}{3}$$

$$\approx 164.67 \ \text{m}$$

15. A ball is thrown directly upward from a point 8 metres above ground with initial velocity $49 m/s$. If acceleration due to gravity is $9.8 m/s^2$, how high will the ball travel?



- We want height of ball
- Aim → find formula for height ie: displacement.

$$\text{accel} = s''(t) = -9.8$$ ← note accel is neg since its in opp direction of travel.

$$\therefore \text{vel} = s'(t) = -9.8t + C$$

Finding C : when $t=0$, $v=49$

$$\therefore \quad 49 = 0 + C$$

$$\therefore \text{vel} = s'(t) = -9.8t + 49$$

$$\therefore \text{disp} = s(t) = \int -9.8t + 49 \, dt$$

$$= -9.8\frac{t^2}{2} + 49t + C_1$$

Finding C_1 : when $t=0$, $s=8$

$$\therefore \quad 8 = 0 + 0 + C_1$$

$$\therefore s(t) = -9.8\frac{t^2}{2} + 49t + 8$$

To find height of ball need to know t when vel $= 0$

ie: $v=0 \rightarrow -9.8t + 49 = 0$

$$t = \frac{49}{9.8} = 5$$

$$\therefore \text{height} = s(5) = -9.8\frac{(5)^2}{2} + 49(5) + 8$$

$$= 130.5 \text{ m.}$$