Integration Rules

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \qquad (n \neq -1)$$

$$\int k \, dx = kx + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int e^x \, dx = e^x + c$$

$$\int \frac{1}{x} \, dx = \log|x| + c$$

eg: 6c) $\int 3e^x - \frac{4}{x} + \frac{1}{x^2} \, dx = \int 3e^x - 4\left(\frac{1}{x}\right) + x^{-2} \, dx$

$$= 3e^x - 4\log|x| + \frac{x^{-1}}{-1} + c$$

$$= 3e^x - 4\log|x| - \frac{1}{x} + c$$

d) $\int \sec^2 x + 2\sin x \, dx = \int \sec^2 x \, dx + 2\int \sin x \, dx$

$= \tan x + 2(-\cos x) + c$

$= \tan x - 2\cos x + c$

e) $\int 5\cos t + 2e^t - \frac{1}{t} \, dt = 5\sin t + 2e^t - \log|t| + c$

f) $\int_1^3 \frac{\sqrt{x} - 1}{x} \, dx = \int_1^3 \frac{\sqrt{x}}{x} - \frac{1}{x} \, dx$

$= \int_1^3 x^{\frac{-1}{2}} - \frac{1}{x} \, dx$

$= \left[2x^{1/2} - \log|x| \right]_1^3$

$= \left[2(3)^{1/2} - \log 3 \right] - \left[2(1)^{1/2} - \log 1 \right]$

$= 2\sqrt{3} - \log 3 - 2$

g) $\int_0^{\pi/2} \cos x - \sin x \, dx = \left[\sin x + \cos x \right]_0^{\pi/2}$

$= \left(\sin \frac{\pi}{2} + \cos \frac{\pi}{2}\right) - (\sin 0 + \cos 0)$

$= (1 + 0) - (0 + 1)$

$= 0$

Remember reverse chain rule:

$$\int (2x+3)^5 \, dx = \frac{1}{2} \times \frac{(2x+3)^6}{6} + C$$

ax+b inside function we know how to integrate

$\frac{1}{a}$ × integ of main fn

This works for other functions too.

eg: $$\int \cos(2x+3) \, dx = \frac{1}{2} \sin(2x+3) + C$$

ax+b inside function we know how to integrate

$\frac{1}{a}$ × Integ of main fn

Infact $$\int \cos(2x) \, dx = \frac{1}{2} \sin 2x + C$$

[check : $\frac{d}{dx}\left(\frac{1}{2}\sin 2x\right) = \frac{1}{2} \cdot 2\cos 2x = \cos 2x$]

ie: $$\int \cos(ax) \, dx = \frac{1}{a} \sin ax$$

$\frac{1}{a}$ × integ of main fn

Similarly $\int \sin ax \, dx = \frac{-1}{a} \cos ax + c$

$$\int e^{ax} \, dx = \underset{\substack{\uparrow \\ \frac{1}{a}}}{\frac{1}{a}} \times \underset{\substack{\uparrow \\ \text{integ of main fn.}}}{e^{ax}} + c$$

eg 7 a) $\int \sin 6x \, dx = \frac{1}{6}(-\cos 6x) + c$

$$= \frac{-1}{6} \cos 6x + c$$

b) $\int \cos 10x \, dx = \frac{1}{10} \sin 10x + c$

c) $\int e^{2x}\,dx = \frac{1}{2}e^{2x} + c$

d) $\int e^{3x}\,dx = \frac{1}{3}e^{3x} + c$

e) $\int e^{-4x}\,dx = \frac{1}{-4}e^{-4x} + c = -\frac{1}{4}e^{-4x} + c$

f) $\int 3\sin 4x\,dx = 3\int \sin 4x\,dx$

$= 3\cdot\frac{1}{4}(-\cos 4x) + c$

$= -\frac{3}{4}\cos 4x + c$

g) $\int \sec^2 3x\,dx = \frac{1}{3}\tan 3x + c$

h) $\int \cos\left(\frac{x}{2}\right)dx = \frac{1}{1/2}\sin\left(\frac{x}{2}\right) + c = 2\sin\frac{x}{2} + c$

i) $\int e^{5x} - \sin 7x\,dx = \frac{1}{5}e^{5x} - \frac{1}{7}(-\cos 7x) + c$

$= \frac{1}{5}e^{5x} + \frac{1}{7}\cos 7x + c$

j) $\int 2\sin 3x + \cos 4x\,dx = \frac{-2}{3}\cos 3x + \frac{1}{4}\sin 4x + c$

This also happens for $ax+b$.

eg: $\frac{d}{dx}\sin(2x+3) = 2\cos(2x+3) \rightarrow \int \cos(2x+3)\,dx = \frac{1}{2}\sin(2x+3) + c$

ie: $\int \sin(ax+b)\,dx = \frac{1}{a}\cos(ax+b) + c$

$\int \cos(ax+b)\,dx = \frac{-1}{a}\sin(ax+b) + c$

$\int e^{ax+b}\,dx = \frac{1}{a}\,e^{ax+b} + c$

$\uparrow$ $\uparrow$

$\frac{1}{a}$ × integ of main fn.

infact: If $F' = f \Rightarrow \int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + c$

Look out for $ax+b$

eg8 a) $\int \sin(3x+4)\,dx = \frac{1}{3}\cos(3x+4) + c$

b) $\int e^{4x-1}\,dx = \frac{1}{4}e^{4x-1} + c$

c) $\int 3\cos(x-1)\,dx = 3\cdot\frac{1}{1}\sin(x-1) + c$

$= 3\sin(x-1) + c$

d) $\int \sec^2(5x+12) = \frac{1}{5}\tan(5x+12) + c$

e) $\int \frac{1}{2x+5}\,dx = \frac{1}{2}\log|2x+5| + c$

f) $\int \frac{1}{(2x+5)^2}\,dx = \int (2x+5)^{-2} + c$

$= \frac{1}{2}\frac{(2x+5)^{-1}}{-1} + c$

$= -\frac{1}{2}(2x+5)^{-1} + c$

g) $\int \frac{4}{3x+2}\,dx = 4\left[\frac{1}{3}\log|3x+2|\right] + c$

$= \frac{4}{3}\log|3x+2| + c$

h) $\int \frac{3}{3x+2}\,dx = 3\cdot\frac{1}{3}\log|3x+2| + c$

$= \log|3x+2| + c$

i) $\int \cos(3-7x)\,dx = -\frac{1}{7}\sin(3-7x) + c$

j) $\int e^{7x} - \sin(1-2x)\,dx = \frac{1}{7}e^{7x} - \left(\frac{1}{-2}(-\cos(1-2x))\right) + c$

$= \frac{1}{7}e^{7x} - \frac{1}{2}\cos(1-2x) + c$

Eg 9.

$$\int_0^2 \frac{2}{1-3x}\,dx$$

$$= 2\int_0^2 \frac{1}{1-3x}\,dx$$

$$= 2 \cdot \frac{-1}{3} \cdot \log|1-3x| \Big|_0^2$$

$$= \frac{-2}{3}\left[\log|1-6| - \log|1-0|\right]$$

$$= \frac{-2}{3}\left[\log|-5| - \log 1\right]$$

$$= \frac{-2}{3}\log 5.$$