Reverse Chain Rule:

$$\int (3x+1)^4 \, dx = \frac{1}{3} \frac{(3x+1)^5}{5} + c$$

Look for $ax+b$ inside a funtion we know how to integrate (ie: powers)

$\frac{1}{a} \times$ integ of main function.

eg) $$\int (5x+2)^7 \, dx = \frac{1}{5} \frac{(5x+2)^8}{8} + c$$

$$= \frac{(5x+2)^8}{40} + c$$

eg) $$\int (x+3)^{50} \, dx = \frac{1}{1} \frac{(x+3)^{51}}{51} + c$$

$1x+3$

$$= \frac{(x+3)^{51}}{51} + c$$

So $\boxed{\int (ax+b)^n \, dx = \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + c}$

Proof: $\frac{d}{dx} (ax+b)^{n+1} = a(n+1)(ax+b)^n$

$\therefore \int a(n+1)(ax+b)^n = (ax+b)^{n+1} + c$

ie: $\int (ax+b)^n \, dx = \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + c$

Egs

3a) $\int (2x+3)^4 \, dx = \frac{1}{2} \frac{(2x+3)^5}{5} + c$

recognise $ax+b$ $\frac{1}{a}$ × integ of "main function"

b) $\int (4x-1)^{10} \, dx = \frac{1}{4} \frac{(4x-1)^{11}}{11} + c$

c) $\int (7-3x)^8 \, dx = \frac{1}{-3} \frac{(7-3x)^9}{9} + c$

$= \frac{-(7-3x)^9}{27} + c$

d) $\int (2-x)^5 \, dx = \frac{1}{-1} \cdot \frac{(2-x)^6}{6} + c$

$= \frac{-(2-x)^6}{6} + c$

e) $\int (2x+3)^{-5}\,dx = \frac{1}{2}\,\frac{(2x+3)^{-4}}{-4} + c$

$= \frac{(2x+3)^{-4}}{-8} + c$

f) $\int \frac{1}{(3x+1)^4}\,dx = \int (3x+1)^{-4}\,dx$

$= \frac{1}{3}\,\frac{(3x+1)^{-3}}{-3} + c$

$= \frac{(3x+1)^{-3}}{-9} + c$

g) $\int (4x-7)^{1/3}\,dx = \frac{1}{4}\,\frac{(\quad)^{4/3}}{4/3} + c$

$= \frac{1}{4}\cdot\frac{3}{4}(4x-7)^{4/3} + c$

$= \frac{3}{16}(4x-7)^{4/3} + c$

h) $\int \sqrt{8x-5}\,dx = \int (8x-5)^{1/2} + c$

$= \frac{1}{8}\,\frac{(8x-5)^{3/2}}{3/2} + c$

$= \frac{1}{\cancel{8}_{4}}\cdot\frac{\cancel{2}}{3}(8x-5)^{3/2} + c$

$= \frac{1}{12}(8x-5)^{3/2} + c$

i) $\int \frac{1}{\sqrt{2x+1}} \, dx = \int (2x+1)^{-1/2} \, dx$

$= \frac{1}{2} \frac{(2x+1)^{1/2}}{1/2} + c$

$= (2x+1)^{1/2} + c$

$= \sqrt{2x+1} + c$

j) $\int \frac{1}{\sqrt{3-7x}} \, dx = \int (3-7x)^{-1/2} \, dx$

$= -\frac{1}{7} \cdot \frac{(3-7x)^{1/2}}{1/2} + c$

$= -\frac{2}{7} (3-7x)^{1/2} + c$

Try Q4

More egs

eg3) a) $\int_1^5 (4x+1)^2\,dx = \left[\frac{1}{4}\,\frac{(4x+1)^3}{3}\right]_1^5$

$$= \frac{1}{4}\left[\frac{(21)^3}{3} - \frac{(5)^3}{3}\right]$$

$$= \frac{1}{4}\left[\frac{9136}{3}\right]$$

$$= \frac{2284}{3}$$

b) $\int_0^2 \frac{3}{(2x-1)^5}\,dx = 3\int_0^2 (2x-1)^{-5}\,dx$

$$= 3\left[\frac{1}{2}\,\frac{(2x-1)^{-4}}{-4}\right]_0^2$$

$$= \frac{-3}{8}\left[(2x-1)^{-4}\right]_0^2$$

$$= \frac{-3}{8}\left[\frac{1}{(2x-1)^4}\right]_0^2$$

$$= \frac{-3}{8}\left[\frac{1}{3^4} - \frac{1}{1^4}\right]$$

$$= \frac{-3}{8}\left[\frac{80}{81}\right]$$

$$= \frac{-10}{27}$$

c) $\int_{-1}^{2} (2-3x)^{-2}\, dx = \left[\frac{-1}{3} \frac{(2-3x)^{-1}}{-1} \right]_{-1}^{2}$

$= \frac{1}{3} \left[\frac{1}{2-3x} \right]_{-1}^{2}$

$= \frac{1}{3} \left[\frac{1}{-4} - \frac{1}{-1} \right]$

$= \frac{1}{3} \left[\frac{3}{4} \right]$

$= \frac{1}{4}$

d) $\int_{3}^{4} \sqrt{x+5}\, dx = \int_{3}^{4} (x+5)^{1/2}\, dx$

$= \left[\frac{1}{1} \frac{(x+5)^{3/2}}{3/2} \right]_{3}^{4}$

$= \frac{2}{3} \left[(x+5)^{3/2} \right]_{3}^{4}$

$= \frac{2}{3} \left[9^{3/2} - 8^{3/2} \right]$

Integrating Other Functions

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$ ← x is the variable, n is a number

$\int k \, dx = kx + c$

$\int \sin x \, dx = -\cos x + c$

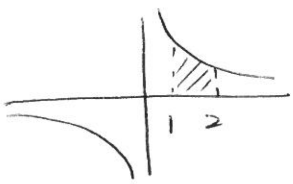
$\int \cos x \, dx = \sin x + c$

$\int e^x \, dx = e^x + c$ ← Here x is the variable + its in the power

$\int \frac{1}{x} \, dx = \log|x| + c$ ← This is the $n=-1$ case

note: Why absolute value?

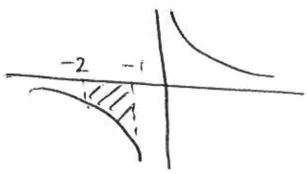
eg: $\int_1^2 \frac{1}{x} \, dx$



$$\int_1^2 \frac{1}{x}\,dx = \log|x| \Big|_1^2 = \log 2 - \log 1 = \log 2$$

$y = \frac{1}{x}$ exists for neg x-values

So $\int_{-2}^{-1} \frac{1}{x} \, dx$



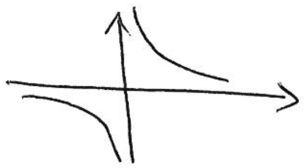
← By symmetry we can see this $= -\log 2$

Algebraically $\int_{-2}^{-1} \frac{1}{x} \, dx = \log|x| \Big|_{-2}^{-1}$

$$= \log|-1| - \log|-2| = -\log 2$$

We use ab value to cater for this case.

Note:



← we can't integrate across an asymptote.

- If we know the derivatives of a funtion, then we also know the integrals.

eg: $\frac{d}{dx}(\cos x) = \sin x \Rightarrow \int \sin x \, dx = \cos x + c$

note: $\int \frac{d}{dx}(\cos x) = \int \sin x \, dx$

↑↑
these
undo each other

Examples

6 a) $\int e^x + \sin x \; dx = e^x - \cos x + c$

b) $\int 3 - \frac{1}{x} \; dx = 3x - \log|x| + c$