Refresher

- Integration = opposite of differentiation

- Fundamental Theorem: If $F' = f$ then $\int_a^b f(x)\,dx = F(b) - F(a)$

- We say F is the anti derivative of f

$$\begin{tikzcd} x^2 \arrow[r, bend left, "\text{diff}"] & 2x \arrow[l, bend left, "\text{integ}"] \end{tikzcd}$$

$$\therefore \int 2x\,dx = x^2 + c$$ ← since anti derivs differ by constant.

- Look at finding anti-derivatives
ie: integration ↗

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + c}$$

eg 1) $\int x^7\,dx = \frac{x^8}{8} + c$

2) $\int x^{-5/6}\,dx = \frac{x^{1/6}}{1/6} + c$

$= 6x^{1/6} + c$

3) $\int 4x \; dx = 4\frac{x^2}{2} + c$

$= 2x^2 + c$

4) $\int 4\sqrt{x} \; dx = 4\int x^{1/2} \; dx$

$= 4\frac{x^{3/2}}{3/2} + c$

$= \frac{8}{3} x^{3/2} + c$

5) $\int \sqrt{4x} \; dx = 2\int x^{1/2} + c$

$= \frac{2x^{3/2}}{3/2} + c$

$= \frac{4}{3} x^{3/2} + c$

6) $\int \frac{3}{\sqrt{x}} \; dx = 3\int x^{-1/2} \; dx$

$= 3\frac{x^{1/2}}{1/2} + c$

$= 6x^{1/2} + c$

Try Q3

More egs

1) $\int 2 \, dx$ ← what differentiates to give 2.

we must have $\int 2 \, dx = 2x + c$

ie! $\boxed{\int k \, dx = kx + c}$ k=constant.

2) $\int 10 \, dx = 10x + c$

3) $\int \sqrt{5} \, dx = \sqrt{5}x + c$

Infact we can add + subtract + these can be integrated separately.

4) $\int 3 - x \, dx = 3x - \frac{x^2}{2} + c$

5) $\int x^4 + 2x \, dx = \frac{x^5}{5} + 2\frac{x^2}{2} + c$

$= \frac{x^5}{5} + x^2 + c$

6) $\int x^{1/3} + 2 \, dx = \frac{x^{4/3}}{4/3} + 2x + c$

$= \frac{3}{4} x^{4/3} + 2x + c$

7) $\int x^3 - x^2 - x - 1 \; dx = \frac{x^4}{4} - \frac{x^3}{3} - \frac{x^2}{2} - x + c$

8) $\int 3x^{-10} - 5x \; dx = 3\left[\frac{x^{-9}}{-9}\right] - 5\left[\frac{x^2}{2}\right] + c$

$= \frac{x^{-9}}{-3} - \frac{5x^2}{2} + c$

9) $\int \sqrt{x} - \frac{1}{\sqrt{x}} \; dx$

$= \int x^{1/2} - x^{-1/2} \; dx$

$= \frac{2}{3}x^{3/2} - 2x^{1/2} + c$

Try Q4

Few more egs.

1) $\int (x+1)(x-3)\, dx$ ← No rule for products

$= \int x^2 - 2x - 3 \, dx$

$= \frac{x^3}{3} - 2\frac{x^2}{2} - 3x + c$

$= \frac{x^3}{3} - x^2 - 3x + c$

2) $\int (2x+1)^2 \, dx$

$= \int 4x^2 + 4x + 1 \, dx$

$= 4\left[\frac{x^3}{3}\right] + 4\left[\frac{x^2}{2}\right] + x + c$

$= \frac{4x^3}{3} + 2x^2 + x + c$

3) $\int \frac{x+1}{x^3} \, dx$ ← No rule for quotients.

$= \int \frac{x}{x^3} + \frac{1}{x^3} \, dx$

$= \int x^{-2} + x^{-3} \, dx$

$= \frac{x^{-1}}{-1} + \frac{x^{-2}}{-2} + c$

$= -\frac{1}{x} - \frac{2}{x^2} + c$

4) $$\int \frac{x-3}{\sqrt{x}} \, dx$$

$$= \int \frac{x}{\sqrt{x}} - \frac{3}{\sqrt{x}} \, dx$$

$$= \int x^{1/2} - 3x^{-1/2} \, dx$$

$$= \frac{2}{3}x^{3/2} - 3\left[2x^{1/2}\right] + c$$

$$= \frac{2}{3}x^{3/2} - 6x^{1/2} + c$$

5) $$\int \frac{3\sqrt{x} - x^2}{2x} \, dx$$

$$= \frac{1}{2}\int \frac{3x^{1/2}}{x} - \frac{x^2}{x} \, dx$$

$$= \frac{1}{2}\int 3x^{-1/2} - x \, dx$$

$$= \frac{1}{2}\left[3\left[\frac{x^{1/2}}{1/2}\right] - \frac{x^2}{2}\right] + c$$

$$= \frac{1}{2}\left[6x^{1/2} - \frac{x^2}{2}\right] + c$$

$$= 3x^{1/2} - \frac{x^2}{4} + c$$

Try Q5