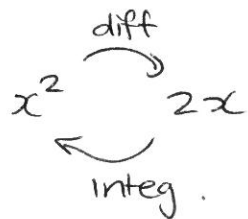


Refresher

• Integration = opposite of differentiation

• Fundamental Theorem: If $F' = f$ then $\int_a^b f(x) dx = F(b) - F(a)$

• We say F is the antiderivative of f



$$\therefore \int 2x dx = x^2 + C \quad \leftarrow \text{since antiderivs differ by constant.}$$

• Look at finding anti-derivatives
ie: integration \nearrow

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

eg 1) $\int x^7 dx = \frac{x^8}{8} + C$

2) $\int x^{-5/6} dx = \frac{x^{1/6}}{1/6} + C$
 $= 6x^{1/6} + C$

$$\begin{aligned} 3) \int 4x \, dx &= 4 \frac{x^2}{2} + C \\ &= 2x^2 + C \end{aligned}$$

$$\begin{aligned} 4) \int 4\sqrt{x} \, dx &= 4 \int x^{1/2} \, dx \\ &= 4 \frac{x^{3/2}}{3/2} + C \\ &= \frac{8}{3} x^{3/2} + C \end{aligned}$$

$$\begin{aligned} 5) \int \sqrt{4x} \, dx &= 2 \int x^{1/2} \, dx \\ &= \frac{2x^{3/2}}{3/2} + C \\ &= \frac{4}{3} x^{3/2} + C \end{aligned}$$

$$\begin{aligned} 6) \int \frac{3}{\sqrt{x}} \, dx &= 3 \int x^{-1/2} \, dx \\ &= 3 \frac{x^{1/2}}{1/2} + C \\ &= 6x^{1/2} + C \end{aligned}$$

Try Q3

More egs

1) $\int 2 \, dx \leftarrow$ what differentiates to give 2.

we must have $\int 2 \, dx = 2x + C$

ie! $\boxed{\int k \, dx = kx + C}$ $k = \text{constant.}$

2) $\int 10 \, dx = 10x + C$

3) $\int \sqrt{5} \, dx = \sqrt{5}x + C$

In fact we can add + subtract + these can be integrated separately.

4) $\int 3 - x \, dx = 3x - \frac{x^2}{2} + C$

5) $\int x^4 + 2x \, dx = \frac{x^5}{5} + 2 \frac{x^2}{2} + C$
 $= \frac{x^5}{5} + x^2 + C$

6) $\int x^{1/3} + 2 \, dx = \frac{x^{4/3}}{4/3} + 2x + C$
 $= \frac{3}{4} x^{4/3} + 2x + C$

$$7) \int x^3 - x^2 - x - 1 \, dx = \frac{x^4}{4} - \frac{x^3}{3} - \frac{x^2}{2} - x + C$$

$$8) \int 3x^{-10} - 5x \, dx = 3 \left[\frac{x^{-9}}{-9} \right] - 5 \left[\frac{x^2}{2} \right] + C$$
$$= \frac{x^{-9}}{-3} - \frac{5x^2}{2} + C$$

$$9) \int \sqrt{x} - \frac{1}{\sqrt{x}} \, dx$$

$$= \int x^{1/2} - x^{-1/2} \, dx$$

$$= \frac{2}{3}x^{3/2} - 2x^{1/2} + C$$

Try Q4

Few more egs.

$$1) \int (x+1)(x-3) dx$$

← No rule for products

$$= \int x^2 - 2x - 3 dx$$

$$= \frac{x^3}{3} - 2 \frac{x^2}{2} - 3x + C$$

$$= \frac{x^3}{3} - x^2 - 3x + C$$

$$2) \int (2x+1)^2 dx$$

$$= \int 4x^2 + 4x + 1 dx$$

$$= 4 \left[\frac{x^3}{3} \right] + 4 \left[\frac{x^2}{2} \right] + x + C$$

$$= \frac{4x^3}{3} + 2x^2 + x + C$$

$$3) \int \frac{x+1}{x^3} dx$$

← No rule for quotients.

$$= \int \frac{x}{x^3} + \frac{1}{x^3} dx$$

$$= \int x^{-2} + x^{-3} dx$$

$$= \frac{x^{-1}}{-1} + \frac{x^{-2}}{-2} + C$$

$$= -\frac{1}{x} - \frac{2}{2x^2} + C$$

$$\begin{aligned}
4) \quad & \int \frac{x-3}{\sqrt{x}} dx \\
&= \int \frac{x}{\sqrt{x}} - \frac{3}{\sqrt{x}} dx \\
&= \int x^{1/2} - 3x^{-1/2} dx \\
&= \frac{2}{3}x^{3/2} - 3 \left[2x^{1/2} \right] + C \\
&= \frac{2}{3}x^{3/2} - 6x^{1/2} + C
\end{aligned}$$

$$\begin{aligned}
5) \quad & \int \frac{3\sqrt{x} - x^2}{2x} dx \\
&= \frac{1}{2} \int \frac{3x^{1/2}}{x} - \frac{x^2}{x} dx \\
&= \frac{1}{2} \int 3x^{-1/2} - x dx \\
&= \frac{1}{2} \left[3 \left[\frac{x^{1/2}}{1/2} \right] - \frac{x^2}{2} \right] + C \\
&= \frac{1}{2} \left[6x^{1/2} - \frac{x^2}{2} \right] + C \\
&= 3x^{1/2} - \frac{x^2}{4} + C
\end{aligned}$$

Try Q5