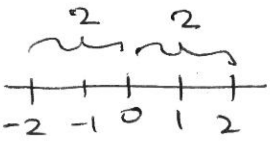
Absolute Value

= distance to 0

$|2|$ = dist of 2 from 0 = 2

$|-2|$ = " " -2 " 0 = 2



So $|2| = 2$, $|-2| = 2$ since dist is always pos

> **Definition**: The absolute value of the real number a is defined by
>
> $$|a| = \begin{cases} a & \text{if } a \geqslant 0 \\ -a & \text{if } a < 0 \end{cases}$$

more formally expressed this way.

So $|-2| = -(-2) = 2$

eg) Write $|1-\pi|$ without absolute value symbols.

$1-\pi$ is neg so $|1-\pi| = -(1-\pi)$

3.14

$= \pi - 1$

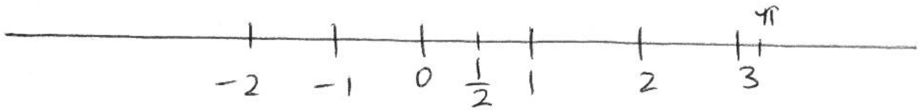
· eg2) $|3-5| = |-2| = 2$ ← treat it like a bracket.

· $|a+b| \neq |a| + |b|$ don't split

$|-3+2| \neq |-3| + |2|$

Infact $|a+b| \leq |a| + |b|$ ← Triangle Inequality.

Types of Numbers



- Signed numbers = positive + negative numbers.
- $\mathbb{N}$ = natural numbers = positive whole numbers
  $= \{1, 2, 3, \cdots\}$
- $\mathbb{Z}$ = integers = pos + neg whole n°s
  $= \{\cdots -2, -1, 0, 1, 2, \cdots\}$
- $\mathbb{Q}$ = rational numbers = fractions

  note: whole n°s are rational since we can write them as fractions $2 = \frac{2}{1}$

- Irrational numbers = numbers that can't be written as fractions.

  eg: $\pi, \sqrt{2}, e$

- $\mathbb{R}$ = real numbers = all numbers on the number line

- we are using set notation to describe real numbers:
  $\mathbb{R}, \mathbb{Z}, \mathbb{Q}, \mathbb{N}$ ← symbols for particular sets of numbers.

$\{ \}$ ← "set of"

$\in$ ← "belongs to"

$2 \in \mathbb{R}$

"2 belongs to real n°s"

$\subset$ ← "is contained in"

$\mathbb{Z} \subset \mathbb{R}$

ie: $\mathbb{Z}$ is a subset of $\mathbb{R}$.

Describing Sets of Numbers

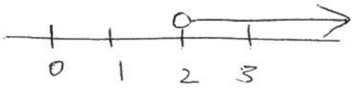
set = collection of objects (here we mainly talk about real numbers)

eg: $\{1,2,3\}$ ← set containing the 3 numbers 1, 2, 3

· Set consisting of numbers $4, -\frac{3}{7}, \sqrt{2}$ is $\{4, -\frac{3}{7}, \sqrt{2}\}$

finitely many objects are listed inside $\{\ \}$

· What about set containing all numbers bigger than 2.



- Use Inequalities to help us describe this.

$>$ $\geqslant$ $<$ $\leqslant$

$>$ greater than

$<$ less than

$5 > 3$ → "5 is greater than 3"

$3 < 5$ → "3 is less than 5"

- We want all numbers > 2.
- Call x our number so we say $x > 2$

(note: not talking abt x in particular, can say $t > 2$ if we like)

∴ In set notation we say $\{x : x > 2\}$

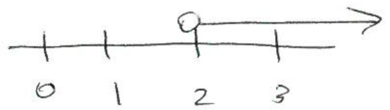
: such that

or $\{x \in \mathbb{R} : x > 2\}$

$\in$ belongs to

: such that

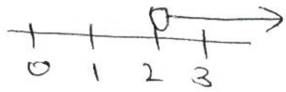
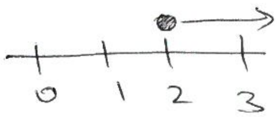
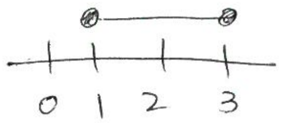
We can also describe this as an interval $(2, \infty)$



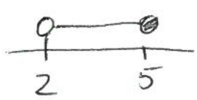
contains all n°s on n° line
between 2 + ∞.

Doesnt include end pts
so we say its an open Interval

So:

Diagram	Ineq	Set notation	Interval
	$x > 2$	$\{x \in \mathbb{R} : x > 2\}$	$(2, \infty)$
	$x \geqslant 2$	$\{x \in \mathbb{R} : x \geqslant 2\}$	$[2, \infty)$
	$1 \leq x \leq 3$	$\{x \in \mathbb{R} : 1 \leq x \leq 3\}$	$[1, 3]$ closed interval

Eg 10: Positive real numbers $= (0, \infty) = \{x \in \mathbb{R} : x > 0\}$

Eg 11: $(2, 5]$  $\{x \in \mathbb{R} : 2 < x \leq 5\}$

dont include 2
do include 5

Indices

ie: raising a number to a power.

$$\underset{\text{base} \rightarrow}{3}^{4 \leftarrow \text{power} = \text{index}} \quad \leftarrow \text{Index Form.}$$

$$3^4 = \underbrace{3 \times 3 \times 3 \times 3}_{\text{4 times.}}$$

$$\boxed{x^4 = x \times x \times x \times x}$$

$3^0 = 1$

$3^1 = 3$

$3^2 = 3 \times 3$

$3^3 = 3 \times 3 \times 3$

$\div 3$ + power reduces by 1

$3^1 = 3$ $\boxed{x^1 = x}$

$3^0 = 1$ Also $2^0 = 1$

$5^0 = 1$

etc

$\boxed{x^0 = 1}$

One step further

$3^{-1} = 1/3$

$3^0 = 1$

$3^1 = 3$

$3^2 = 3 \times 3$

$$3^{-1} = \frac{1}{3}$$

Infact $2^{-1} = \frac{1}{2}$

$6^{-1} = \frac{1}{6}$

$$\boxed{x^{-1} = \frac{1}{x}}$$

eg) $3^{-1} = \frac{1}{3}$

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

Fractional powers → roots.

$5^{1/2} = \sqrt{5}$

$5^{1/3} = \sqrt[3]{5}$

$5^{1/4} = \sqrt[4]{5}$

eg 1) $4^{1/2} = \sqrt{4} = 2$

2) $27^{1/3} = \sqrt[3]{27} = 3$

Some Rules

Multiplying : eg: $3^2 \times 3^3 = 3 \times 3 \times 3 \times 3 \times 3$

$= 3^5 \leftarrow 2+3$

$\boxed{x^a \times x^b = x^{a+b}}$ ← bases must be same

Dividing : eg: $\frac{3^5}{3^3} = \frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3} = 3 \times 3 = 3^2$

$\boxed{\frac{x^a}{x^b} = x^{a-b}}$ ← same bases subtract power

Powers: $(3^2)^3 = 3^2 \times 3^2 \times 3^2$

$= 3\times3 \times 3\times3 \times 3\times3$

$= 3^6$

$$\boxed{(x^a)^b = x^{a\times b}}$$

egs 1) $2^2 \times 2^3 = 2^5$

2) $2^2 \times 3^2 = 4\times 9 = 36$ (This is also $(2\times3)^2 = 6^2 = 36$)

3) $2^2 + 3^2 = 4+9 = 13$ (This is NOT 5^2)

summary:

- $x^a \times x^b = x^{a+b}$
- $\dfrac{x^a}{x^b} = x^{a-b}$
- $(x^a)^b = x^{a\times b}$
- $x^1 = x$
- $x^0 = 1$
- $x^{-1} = \dfrac{1}{x}$
- fractional powers → roots.

Note: Mult rule forces us to define the "unusual" powers.

$2^3 = 2^{3+0} = 2^3 \times 2^0 \Rightarrow 2^0 = 1$

$2^{-1} \times 2^4 = 2^3 \Rightarrow 2^{-1} = \frac{1}{2}$

$2^{1/2} \times 2^{1/2} = 2^{1/2+1/2} = 2 \Rightarrow 2^{1/2} = \sqrt{2}$.

Eg 12

a) $(5^{4})^{-2} \times 5^{3} \div 5 = 5^{-8} \times 5^{3} \div 5^{1}$

$= 5^{-8+3} \div 5^{1}$

$= 5^{-5} \div 5^{1}$

$= 5^{-5-1}$

$= 5^{-6}$

$= \frac{1}{5^{6}}$

b) $16^{-1/2} = \frac{1}{16^{1/2}} = \frac{1}{4}$

c) $(16^{1/2})^{2} = 16$

d) $27^{-2/3} = \frac{1}{27^{2/3}} = \frac{1}{(27^{1/3})^{2}} = \frac{1}{3^{2}} = \frac{1}{9}$

e) $\sqrt{8} \times \sqrt{2} = 8^{1/2} \times 2^{1/2}$

$= (2^{3})^{1/2} \times 2^{1/2}$

$= 2^{3/2} \times 2^{1/2}$

$= 2^{2}$

f) $7^{2} \div 7^{-4} \times 3 \times \sqrt{81} = 7^{2-(-4)} \times 3 \times 81^{1/2}$

$= 7^{6} \times 3^{1} \times (3^{4})^{1/2}$

$= 7^{6} \times 3^{1} \times 3^{2}$

$= 7^{6} \times 3^{2}$

Note! Now that we know about neg powers we can write more scientific notation $0.00834 = 8.34 \times 10^{-3}$