Ratio + Proportion

<u>ratio</u> compares the sizes of 2 things.

Eg 7: class has 20 males + 24 females.

a) Ratio of males to females is 20:24 or $\frac{20}{24} = \frac{5}{6}$

usually write in simplest form 5:6

b) females to males is 6:5

<u>proportion</u> = ratio of n° of items to total

c) Prop of females in class = $\frac{24}{20+44}$ ← n° of females ← total.

$= \frac{24}{44}$

$= \frac{6}{11}$

d) Prop of males in class = $\frac{20}{44} = \frac{5}{11} = 0.45$ ← Sometimes written as a decimal.

e) Percentage of males in class: $\frac{5}{11}$ were male

∴ want this fraction out of 100%

ie: $\frac{5}{11} \times 100\% = 45\%$

Another use of the word proportion:

consider Photos:

4×6 $\quad$ 8×12 $\quad$ 10×15 $\quad$ 12×18

Notice the length : width ratios are the same.

$6:4$ $\qquad$ $12:8$

$= 3:2$ $\qquad$ $3:2$

- We say the photos are <u>similar</u> to each other.
- Since Length : width ratios are same we say

  Length is proportional to width

  $L \propto W$ ← symbol for proportionality

  Here $L = 1.5 \times W$

  We say 1.5 is the <u>constant of proportionality</u>

<u>Definition</u>

If A and B are proportional then $A = kB$

If " " " " inversely proportional then $A = \frac{k}{B}$

Unit Conversion

is an example of when things are in proportion

eg: $1m = 100 cm$ ← m and cm are proportional (const of prop = 100).

eg: Convert 20 m/s to kilometers per hour.

We know $1 km = 1000m$

$1 hr = 60 mins = 60 \times 60$ secs

$= 3600$ secs

Start with 20 m/s

same as 0.02 km/s ← If 1000 m = 1 km, Then 10 m = 0.01 kms, 20 m = 0.02 kms

$\therefore 0.02 \times 3600 / 3600 s$ ← 1 hr = 3600 secs

72 km / 1 hr.

Square Roots

= opposite of squaring

$3^2 = 9$ so $\sqrt{9} = 3$

$4^2 = 16$ so $\sqrt{16} = 4$

$5^2 = 25$ so $\sqrt{25} = 5$

$6^2 = 36$ so $\sqrt{36} = 6$

⋮

9, 16, 25 ... are called perfect squares.

$\sqrt{\ }$ ← radical sign (surd)

- What about $\sqrt{2}$
  - 2 is not a perfect square.
  - Infact on calc $\sqrt{2} = 1.414213\ldots$ ← goes forever.
    - it can't be written as a fraction
  - Leave $\sqrt{2}$ as $\sqrt{2}$ ← in its exact form.

$\boxed{\text{Defn: } \sqrt{a} = b \text{ means } b^2 = a \text{ AND } b \geq 0}$

ie: $\sqrt{9} = 3$ since $3^2 = 9$ and $3 > 0$

So $\sqrt{\ }$ means "the positive square root of"

- we can't take $\sqrt{\ }$ of a neg.

nth root : $\sqrt[n]{a} = b$ means $b^n = a$.

eg: $\sqrt[2]{9} = 3$ means $3^2 = 9$

$\sqrt[3]{8} = 2$ means $2^3 = 8$

$\sqrt[4]{81} = 3$ means $3^4 = 81$

$\sqrt[3]{-8} = -2$ means $(-2)^3 = -8$

Working with sq Roots

eqs) $\sqrt{3} \times \sqrt{5} = \sqrt{3\times 5} = \sqrt{15}$

$\sqrt{3} + \sqrt{5} \leftarrow$ can't simplify further

$\sqrt{3} + 2\sqrt{3} = 3\sqrt{3}$ $\leftarrow$ collect like surds.

$(\sqrt{3})^2 = 3$

$(\sqrt{3})^3 = \sqrt{3}\times\sqrt{3}\times\sqrt{3} = 3\sqrt{3}$

$$\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$$

$$\sqrt{\frac{3}{100}} = \frac{\sqrt{3}}{\sqrt{100}} = \frac{\sqrt{3}}{10}$$

Some Rules we have been following:

- $\sqrt{a}\times\sqrt{b} = \sqrt{a\times b}$
- $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$
- $(\sqrt{a})^2 = a$
- $\sqrt{0} = 0$
- $\sqrt{neg}$ = undefined.
- No rule for +/- .

Look out for perfect squares:

$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$

So $\dfrac{\sqrt{8}}{\sqrt{2}} = \dfrac{2\sqrt{2}}{\sqrt{2}} = 2$

Eg 8) Simplify a) $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$

b) $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$

c) $\sqrt{8} + \sqrt{2} = 2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$

d) $\sqrt{2}\,(4 + \sqrt{3}) = 4\sqrt{2} + \sqrt{2} \cdot \sqrt{3}$

$= 4\sqrt{2} + \sqrt{6}$

e) $\sqrt{8 - -3 \times 4} = \sqrt{8 + 3 \times 4} = \sqrt{8 + 12} = \sqrt{20} = 2\sqrt{5}$

<u>Rationalising the Denominator</u>

eg) $\dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2}{\sqrt{2}}$

"1 in disguise"

f) $\dfrac{4}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{4\sqrt{3}}{3}$