A few more tricks.

Differentiate the following functions.

4 a) $y = (x+2)(x+3)$

$= x^2 + 5x + 6$

$\frac{dy}{dx} = 2x + 5$

b) $y = \frac{x^3 + 5}{x}$

$= \frac{x^3}{x} + \frac{5}{x}$

$= x^2 + 5x^{-1}$

$y' = 2x - 5x^{-2}$

c) $f(x) = (x - \sqrt{x})^2$

$= x^2 - 2x\sqrt{x} + x$

$= x^2 - 2x^{3/2} + x$

$f'(x) = 2x - 2\left(\frac{3}{2}x^{1/2}\right) + 1$

$= 2x - 3x^{1/2} + 1$

d) $f(x) = \frac{1 + \sqrt{x}}{x^2}$

$= \frac{1}{x^2} + \frac{\sqrt{x}}{x^2} = x^{-2} + x^{-3/2}$

$\frac{df}{dx} = -2x^{-3} - \frac{3}{2}x^{-5/2}$

Eg5) $f(x) = x^4 + x^2 + 1$. Find $f'(2)$

we are given the function + we want the derivative at the point $x=2$

First find the derivative

$$f'(x) = 4x^3 + 2x$$

$\therefore$ At $x=2$, $f(x) = 4(2)^3 + 2(2)$

$$= 36$$

Eg6) $f(x) = 3\sqrt{x} - \frac{4}{x^2}$ Find $f'(1)$

ie: $f(x) = 3x^{1/2} - 4x^{-2}$

$$\therefore f'(x) = 3\left(\tfrac{1}{2}x^{-1/2}\right) - 4\left(-2x^{-3}\right)$$

$$= \frac{3}{2}x^{-1/2} + 8x^{-3}$$

$$= \frac{3}{2\sqrt{x}} + \frac{8}{x^3}$$

$$f'(1) = \frac{3}{2} + 8$$

$$= \frac{19}{2}$$

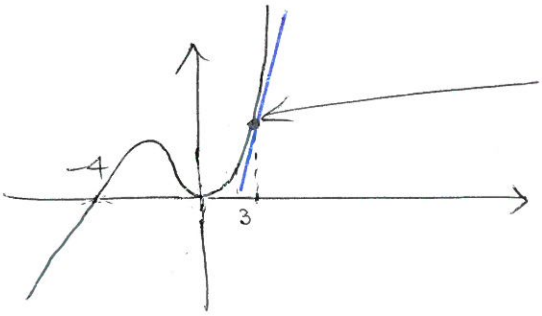
Eg7) $f(x) = x^3 + 4x^2$. Find $f'(3)$

$$f'(x) = 3x^2 + 8x$$

$$\therefore f'(3) = 3(3)^2 + 8(3)$$
$$= 27 + 24$$
$$= 51$$

Recall what this means

$f(x) = x^3 + 4x^2$
$= x^2(x+4)$



$f'(3) = 51$

slope at $x = 3$

Actually its the slope of the tangent to the curve at $x = 3$.

So we can find the eqn of this tangent. at $x = 3$

Tangent = Line $y - y_1 = m(x - x_1)$

Need m = slope $= f'(3) = 51$

point (x_1, y_1) : $x = 3$ so $\underset{f(3)}{y} = 3^3 + 4(3)^2 = 63$

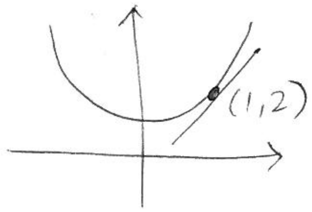
$\therefore$ Tang is $y - 63 = 51(x - 3)$

ie: $y - 63 = 51x - 153$

$y = 51x - 90$

Eg8) Find the equation of the tangent to the curve
$f(x) = 3x^2 - 2x + 1$ at the point $(1,2)$

Imagine whats happenning



Want tangent : $y - y_1 = m(x - x_1)$ $\qquad (x_1, y_1) = (1,2)$

Need m = slope = derivative at $x = 1$

$\therefore f'(x) = 6x - 2$

$f'(1) = 6 - 2 = 4$

$\therefore m = 4$

$\therefore$ tangent is $y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y = 4x - 2$

Eg9) Find the equation of the tangent to the curve $y = \frac{1}{\sqrt{x}}$ at $x = 4$.

Tangent : $y - y_1 = m(x - x_1)$

$x_1 = 4 \Rightarrow \therefore y_1 = \frac{1}{\sqrt{4}} = \frac{1}{2}$ $\quad \therefore (x_1, y_1) = (4, \frac{1}{2})$

m = slope = derivative at $x = 4$

$$y = \frac{1}{\sqrt{x}} = x^{-1/2}$$

$$\frac{dy}{dx} = \frac{-1}{2} x^{-3/2}$$

At $x = 4$: $\frac{dy}{dx} = -\frac{1}{2}(4)^{-3/2}$

$$= -\frac{1}{2}\left(\frac{1}{2^3}\right)$$

$$= -\frac{1}{16}$$

$$\therefore m = -\frac{1}{16}$$

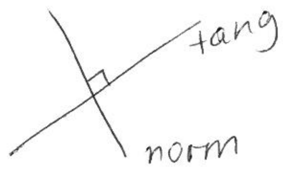
$\therefore$ Tangent is $y - \frac{1}{2} = -\frac{1}{16}(x - 4)$

$$y - \frac{1}{2} = -\frac{1}{16}x + \frac{1}{4}$$

$$y = -\frac{1}{16}x + \frac{3}{4}$$

Normals

If we know the tangent we can find the normal



normal = line perpendicular to tangent
(remember gradients of perp lines multiply to give -1)

Eg10) Find the equation of the normal to the curve $y = \frac{1}{\sqrt{x}}$ at $x = 4$.

We saw point $= (4, \frac{1}{2})$ + gradient of tang $= -\frac{1}{16}$

$\therefore$ gradient of normal $= 16$

Normal is
$$y - y_1 = m(x - x_1)$$
$$y - \frac{1}{2} = 16(x - 4)$$
$$y - \frac{1}{2} = 16x - 64$$
$$y = 16x - \frac{127}{2}$$

Eg11) Find the equations of the tangent and normal to the curve $y=\sqrt{x}$ at the point $(9,3)$

Tangent : $y-y_1 = m(x-x_1)$

. $(x_1, y_1) = (9,3)$

. m = slope = derivative at $x=9$

$$y = \sqrt{x} = x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

at $x=9$, $\frac{dy}{dx} = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

$\therefore$ Tangent is $y-3 = \frac{1}{6}(x-9)$

$$y-3 = \frac{1}{6}x - \frac{3}{2}$$

$$y = \frac{1}{6}x + \frac{3}{2}$$

Normal : $y-y_1 = m(x-x_1)$

. $(x_1, y_1) = (9,3)$

. m = slope of norm so $m \times \frac{1}{6} = -1$

$\therefore m = -6$

$\therefore$ Normal is $y-3 = -6(x-9)$

$$y-3 = -6x+54$$

$$y = -6x+57$$

ie: