We actually have some rules to make differentiation a lot simpler.

$$\boxed{\begin{aligned} f(x) &= x^n \\ f'(x) &= nx^{n-1} \end{aligned}}$$

Recall: $f(x) = x^2$ & $f'(x) = 2x$ ← we've seen these.

$f(x) = x^3$ & $f'(x) = 3x^2$ ←

Proof: $f(x) = x^n$

so $f(x+h) = (x+h)^n = (x+h)(x+h) \ldots (x+h)$

$= x^n + nx^{n-1}h + \ldots + h^n$

terms with x's & hs

$$\therefore f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{x^n + nx^{n-1}h + (x^k h^k) + \ldots + h^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(nx^{n-1} + hx^{n-2}(\ldots) + \ldots + h^{n-1})}{\cancel{h}}$$

$$= nx^{n-1}$$

Eg a) $y = x^5$

$$\frac{dy}{dx} = 5x^4$$

b) $f(x) = x^{10}$

$f'(x) = 10x^9$

c) $y = x^{-3}$

$y' = -3x^{-4}$

d) $y = \frac{1}{x^5}$

$= x^{-5}$

$$\frac{dy}{dx} = -5x^{-6}$$

e) $y = x^{1/3}$

$y' = \frac{1}{3}x^{-2/3}$

f) $y = \sqrt{x} = x^{1/2}$

$y' = \frac{1}{2}x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$

g) $y = \frac{1}{x} = x^{-1}$

$y' = -x^{-2}$ ← we saw this when doing first principles

Try Q2

Infact we can say:

$y = f(x)$	$y' = f'(x) = \frac{dy}{dx}$
x^n	nx^{n-1}
constant k	0
$k\,f(x)$	$k\,f'(x)$
$f(x) \pm g(x)$	$f'(x) \pm g'(x)$

Eg a) $f(x) = 4x^2$

$f'(x) = 4(2x) = 8x$

b) $f(x) = 4x^2 + 1$ ← we saw this one earlier

$= 8x + 0 = 8x$

c) $y = x^2 + 3x + 2$

$\frac{dy}{dx} = 2x + 3$ ← we saw this one earlier

d) $y = x + \sqrt{x}$

$= x + x^{1/2}$

$y' = 1 + \frac{1}{2}x^{-1/2}$

e) $y = 5x^2 + \frac{1}{x}$

$= 5x^2 + x^{-1}$

$\frac{dy}{dx} = 10x - x^{-2}$

f) $f(x) = 3x^6 - \sqrt{x} + \frac{1}{x^2}$

$= 3x^6 - x^{1/2} + x^{-2}$

$f'(x) = 18x^5 - \frac{1}{2}x^{-1/2} - 2x^{-3}$

$= 18x^5 - \frac{1}{2\sqrt{x}} - \frac{2}{x^3}$

g) $g(x) = \frac{1}{4x^2} + \pi$

$= \frac{1}{4}x^{-2} + \pi$

$g'(x) = \frac{1}{4}(-2x^{-3}) + 0$

$= -\frac{1}{2}x^{-3}$

h) $f(t) = 50 - \sqrt{2t} + \frac{1}{\sqrt{t}}$

$= 50 - \sqrt{2}\,t^{1/2} + t^{-1/2}$

$f'(t) = 0 - \sqrt{2}\left(\frac{1}{2}t^{-1/2}\right) + \left(-\frac{1}{2}t^{-3/2}\right)$

$= -\frac{\sqrt{2}}{2}t^{-1/2} - \frac{1}{2}t^{-3/2}$

i) $y(m) = m^{17} - \frac{8}{\sqrt{m}}$

$= m^{17} - 8m^{-1/2}$

$\frac{dy}{dm} = 17m^{16} - 8\left(-\frac{1}{2}m^{-3/2}\right)$

$= 17m^{16} + 4m^{-3/2}$

Try Q3