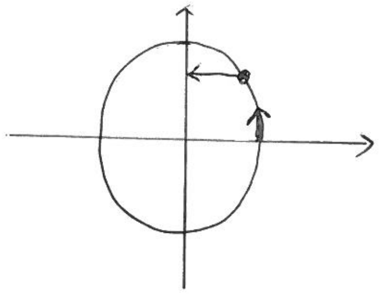
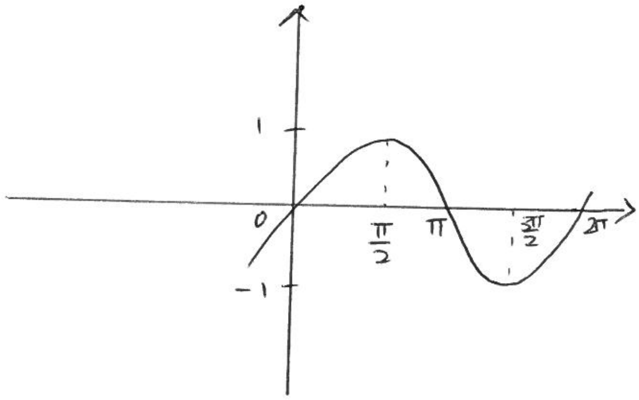
Travelling around the unit circle

In fact as we travel around the unit circle, look at what the y-values are doing



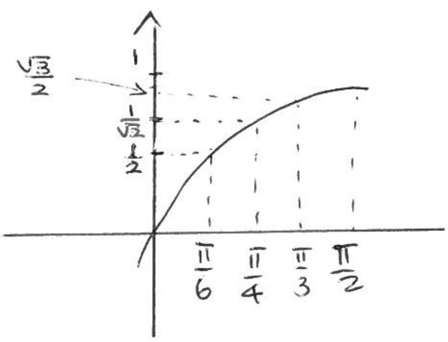
- start at 0
- go up to 1 at $\frac{\pi}{2}$
- down to 0 at π
- down to -1 at $\frac{3\pi}{2}$
- back to 0 at 2π

we can graph this:

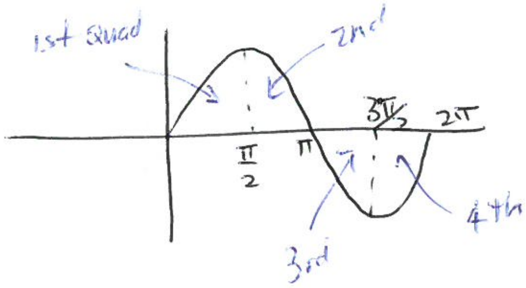


← The graph of $y = \sin x$

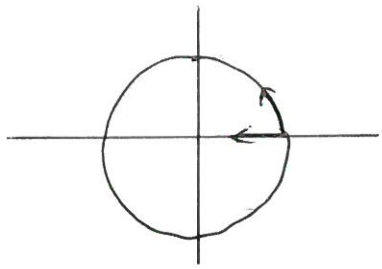
- This graph keeps going since we can keep going round + round our circle
- The values on our graph tell us the same thing as our unit circle.

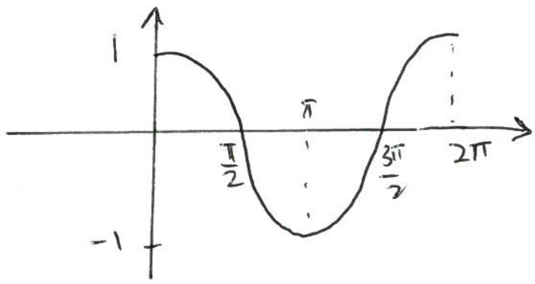


Also notice we can see quadrants



We can do something similar for cos ie: x-values



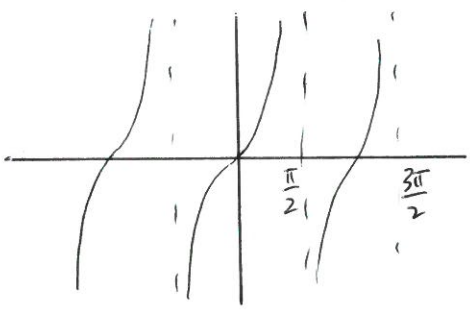


Infact cosine = complement of sine

ie: $\cos x = \sin\left(\frac{\pi}{2} - x\right)$

$\cos x$ = sin of complement of x

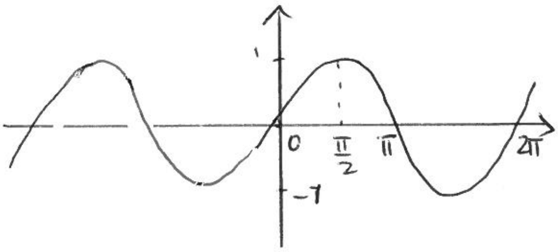
For $y = \tan x$ → use $\tan x = \frac{\sin x}{\cos x}$



$\tan x$ is not defined when $\cos x = 0$

Trig Functions

$y = \sin x$



Notice: • Every 2π we get the same thing repeating

→ Period $= 2\pi$

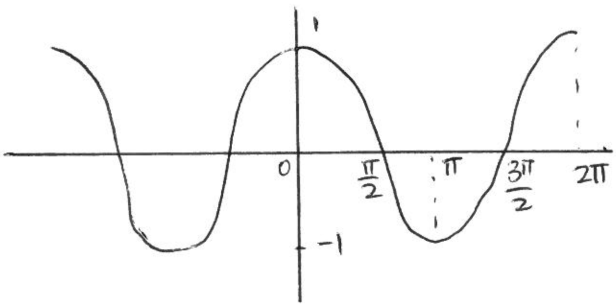
• Range → $-1 \leq \sin x \leq 1$

• This is an odd function → $\boxed{\sin(-x) = -\sin x}$

• We can read off values

$\sin 0 = 0$, $\sin \frac{\pi}{2} = 1$, $\sin \pi = 0 = \sin 2\pi = \sin 3\pi$

$y = \cos x$



Notice • Period $= 2\pi$

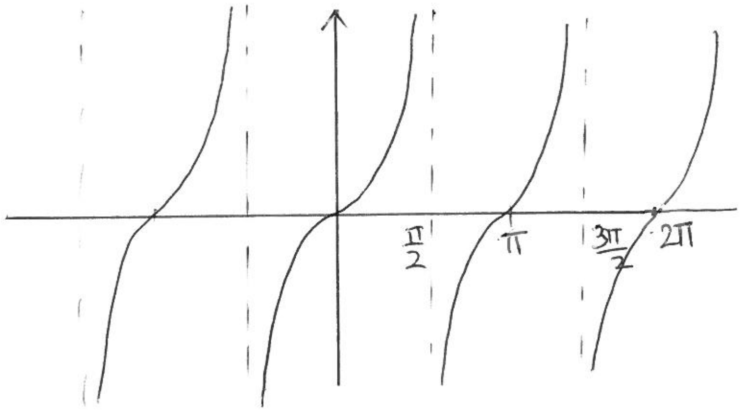
• $-1 \leq \cos x \leq 1$

• This is an even function → $\boxed{\cos(-x) = \cos x}$

• Reading off values :

$\cos 0 = 1$, $\cos \frac{\pi}{2} = 0$, $\cos \pi = -1$

$y = \tan x$



Notice : • Period $= \pi$

• tan is undefined for $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots -\frac{\pi}{2}, -\frac{3\pi}{2} \ldots$

• We can read some values

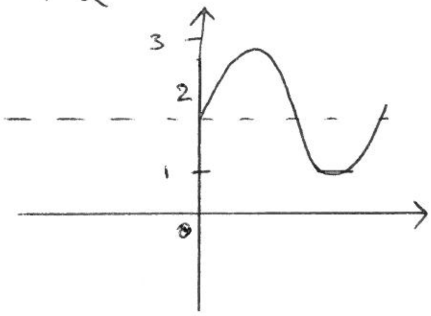
$\tan 0 = 0 = \tan \pi = \tan 2\pi$

• $\tan x$ is an odd function → $\boxed{\tan(-x) = -\tan x}$

- Now we can think of $\sin x$, $\cos x$ and $\tan x$ as functions.
- Remember all the things we did with functions
  - → Domain + Range (Input + Output)
  - → Odd + Even
  - → Increasing + Decreasing
  - → Reading off info from graphs
  - → Modifying graphs

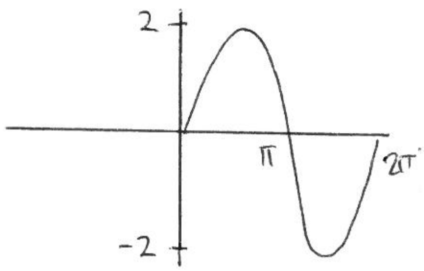
Modifying Trig Functions

eg 1) $y = \sin x + 2$



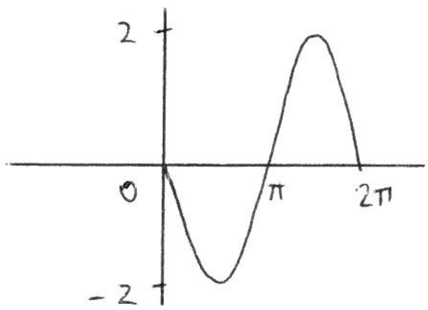
vertical shift

2) $y = 2\sin x$



stretch
amplitude

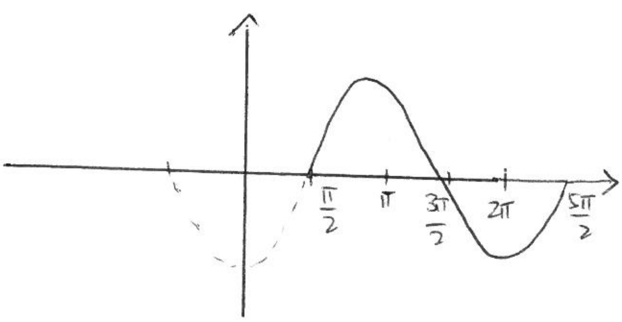
3) $y = -2\sin x$



amplitude = 2

4) $y = \sin\left(x - \frac{\pi}{2}\right)$ ← horizontal shift to right

ie: phase shift

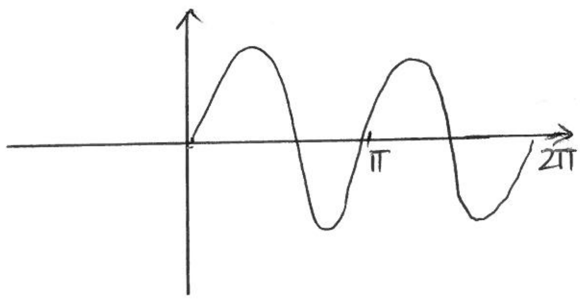


5) $y = \sin 2x$

affects period.

For $\sin x$ ← period is 2π

For $\sin 2x$ ← we get to 2π in half the time



The number in front of x affects the period.

Period of $y = \sin kx$ is $\dfrac{2\pi}{k}$

In general we write $y = A \sin k(x-a)$

amplitude

affects period

phase shift

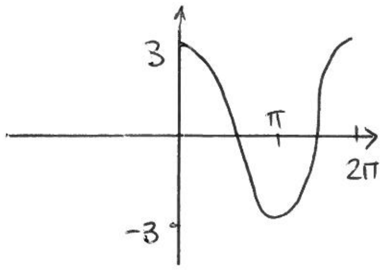
Similarly we can do the same of $\cos x$ and $\tan x$

ie: $y = A \cos k(x-a)$ ← period $= \dfrac{2\pi}{k}$

$y = A \tan k(x-a)$ ← period $= \dfrac{\pi}{k}$

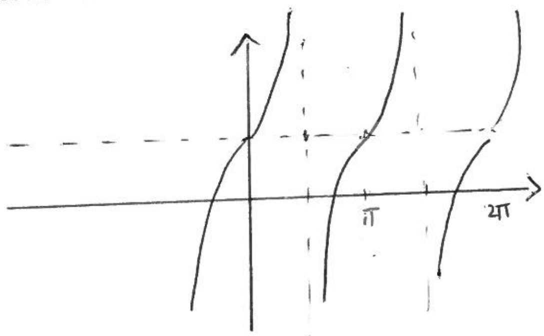
Eg 4) Sketch the following functions.

a) $y = 3\cos x$



Amplitude $= 3$
Period $= 2\pi$

b) $y = 1 + \tan x$

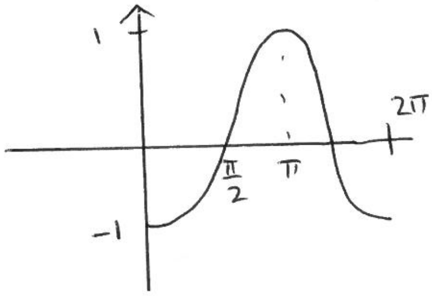


vertical shift

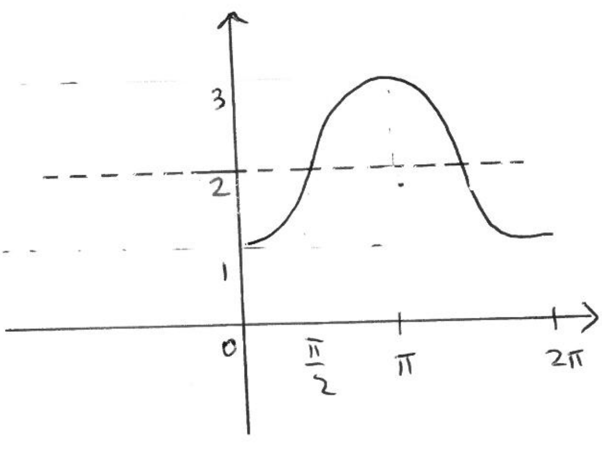
Period $= \pi$

c) $y = 2 - \cos x$

Firstly
$y = -\cos x$

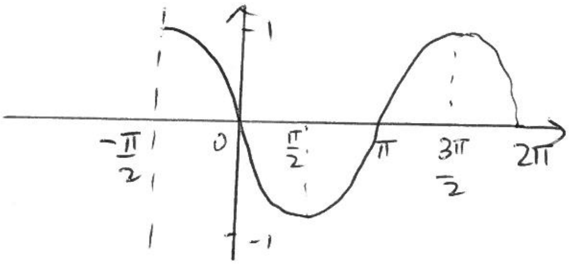


$\therefore$ $y = 2 - \cos x$ shifts this up 2 units



d) $y = \cos\left(x + \frac{\pi}{2}\right)$

← horizontal shift

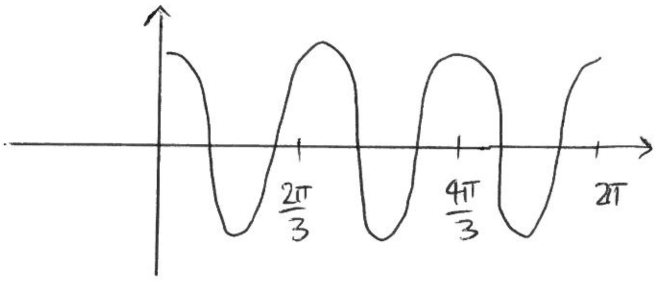


Amplitude = 1

Period = 2π

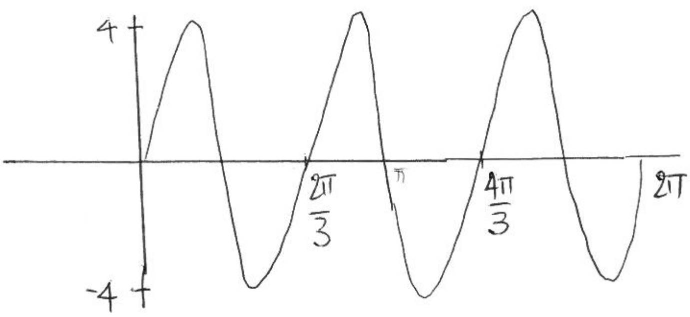
e) $y = \cos 3x$

← period = $\frac{2\pi}{3}$

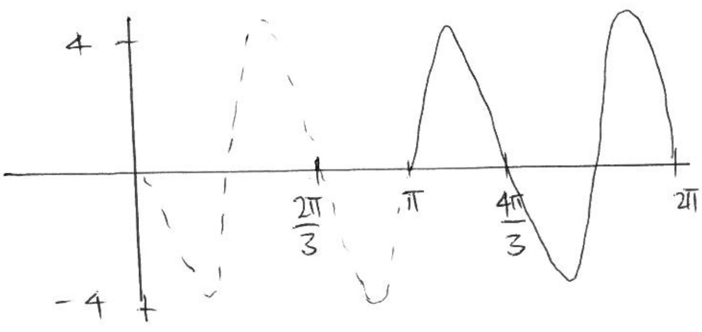


Amp = 1

f) $y = 4\sin 3x$



g) $y = 4\sin 3(x - \pi)$ ← now shift to right by π

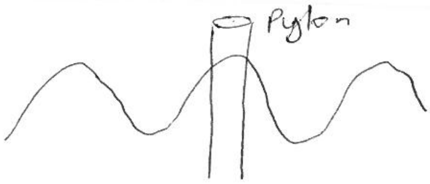


Eg

5. As a wave passes by an offshore pylon, the height of the water is modelled by the function $h(t) = 3\cos\left(\frac{\pi}{10}t\right)$, where $h(t)$ is the height in metres above the mean sea level at time t.

(a) Find the period of the wave.

(b) Find the wave height (ie. the vertical distance between the trough and the crest of the wave).

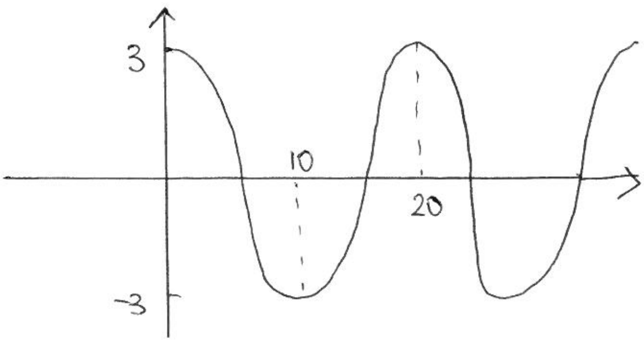


$$h(t) = 3\cos\frac{\pi}{10}t.$$

a) Period $= \dfrac{2\pi}{\pi/10} = \dfrac{20\pi}{\pi} = 20$

↳ ie: one cos curve in 20 units.

We can picture whats happenning!



$(A = 3)$

b) Wave height $= 6\text{m}$