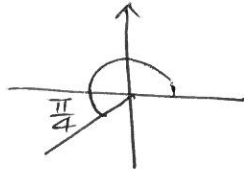


## Trig

We know

- Right angled triangles + trig ratios
- Radians + drawing angles
- Unit Circle  $\rightarrow$  ASTC
- Exact values

Revision! eg: Find the exact value of  $\cos \frac{5\pi}{4}$



- Draw
- 3rd quad - cos is neg
- $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$\therefore \cos \frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$$

$\nearrow$

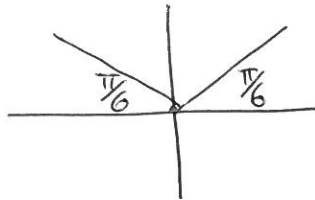
We have been looking at questions where we are given angle + we want exact value.

Now we look at: Given exact value, what is the angle:

eg:  $\sin \theta = \frac{1}{2}$  . Find  $\theta$  .

- Positive  $\rightarrow$  1st or 2nd quad
- $\sin \frac{\pi}{6} = \frac{1}{2}$  so  $\frac{\pi}{6}$  = angle we make with x-axis

• Draw



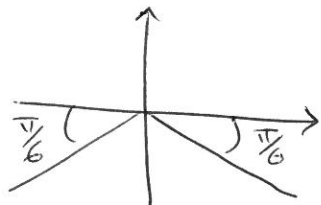
$$\therefore \theta = \frac{\pi}{6}$$

$$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Eg For each of the following trig eqns, find  $\theta$  for  $0 \leq \theta < 2\pi$ .

a)  $\sin \theta = -\frac{1}{2}$

• neg  $\rightarrow$    $\sin \frac{\pi}{6} = \frac{1}{2}$



$$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

$$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

Note:  
This means



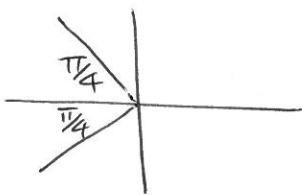
Steps:

1. Pos/Neg  $\rightarrow$  quadrant
2. Value  $\rightarrow$  Angle made with x-axis
3. Draw

b)  $\cos \theta = -\frac{1}{\sqrt{2}}$

• neg  $\rightarrow$  cos is neg in 2nd + 3rd quad

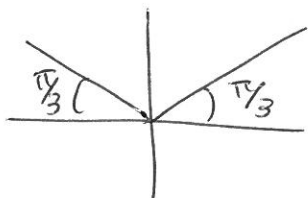
•  $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \rightarrow \frac{\pi}{4}$  is angle



$$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

c)  $\sin \theta = \frac{\sqrt{3}}{2}$



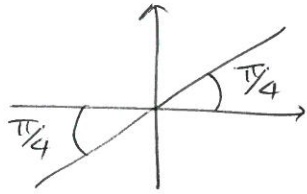
$$\theta = \frac{\pi}{3}$$

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

d)  $\tan \theta = 1$

• Pos  $\rightarrow$  tan is pos  $\begin{matrix} \checkmark \\ + \\ \checkmark \end{matrix}$  in 1st + 3rd quad

•  $\tan \frac{\pi}{4} = 1 \rightarrow \frac{\pi}{4}$  is the angle



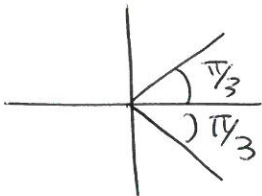
$$\theta = \frac{\pi}{4}$$

$$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

e)  $\cos \theta = \frac{1}{2}$

• Pos  $\rightarrow$  cos is pos  $\begin{matrix} \checkmark \\ + \\ \checkmark \end{matrix}$  in 1st + 4th quad

•  $\cos \frac{\pi}{3} = \frac{1}{2} \rightarrow \frac{\pi}{3}$  is the angle



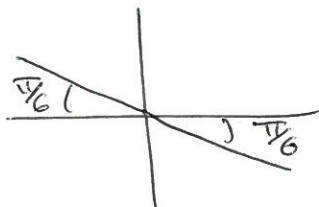
$$\theta = \frac{\pi}{3}$$

$$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

f)  $\tan \theta = -\frac{1}{\sqrt{3}}$

• Neg  $\rightarrow$  tan is neg  $\begin{matrix} \checkmark \\ - \\ \checkmark \end{matrix}$  in 2nd + 4th quad.

•  $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \rightarrow \frac{\pi}{6}$  is the angle



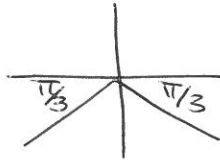
$$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

2 a)  $\sin x = -\frac{\sqrt{3}}{2}$   $0 \leq x \leq 2\pi$

• neg  $\rightarrow$  3rd + 4th quad

•  $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

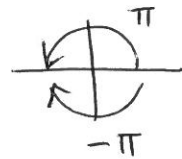


$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

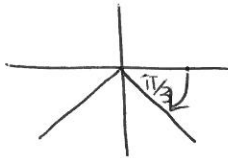
$x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

b)  $\sin x = -\frac{\sqrt{3}}{2}$   $-\pi \leq x \leq \pi$

$\uparrow$  Now looking at



so



$x = -\frac{\pi}{3}$

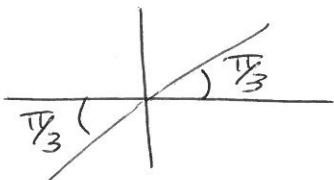
$x = -\frac{2\pi}{3}$

c)  $\tan x = \sqrt{3}$   $0 \leq x \leq 4\pi$

$\curvearrowright$  Now going round twice.

• Pos  $\rightarrow$  1st, 3rd quad

•  $\tan \frac{\pi}{3} = \sqrt{3}$



$x = \frac{\pi}{3}$

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

} 1st revolution

also  $x = 2\pi + \frac{\pi}{3} = \frac{7\pi}{3}$

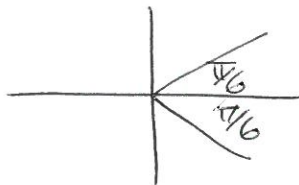
$x = 2\pi + \frac{4\pi}{3} = \frac{10\pi}{3}$

} 2nd revolution

$$d) \cos x = \frac{\sqrt{3}}{2} \quad -\pi \leq x \leq \pi$$

Pos  $\rightarrow$  1st + 4th quad

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$



$$\therefore x = \frac{\pi}{6}, -\frac{\pi}{6}$$

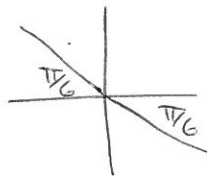
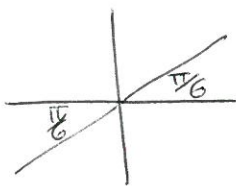
$$3) \text{ Solve } \tan^2 x = \frac{1}{3} \quad 0 \leq x \leq 2\pi$$

$$\text{ie: } \tan x = \pm \frac{1}{\sqrt{3}}$$

$$\left( \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \right)$$

$$\tan x = \frac{1}{\sqrt{3}}$$

$$\tan x = -\frac{1}{\sqrt{3}}$$



$$x = \frac{\pi}{6}$$

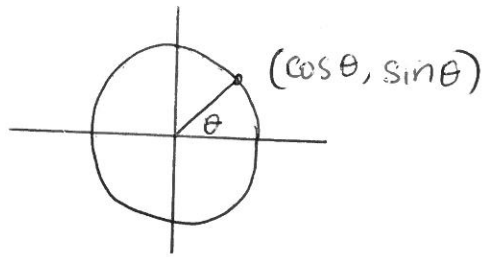
$$x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

$$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

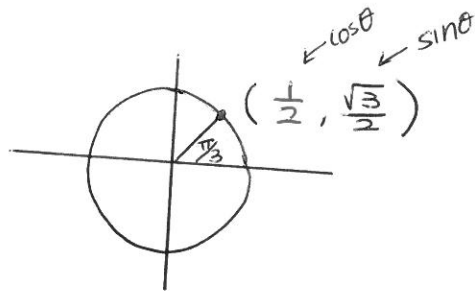
$$x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

# more Exact Values

Remember

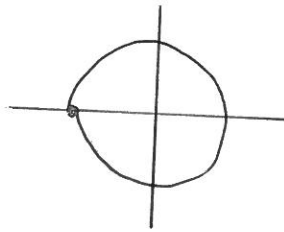


$$\sin \theta = y\text{-values}$$
$$\cos \theta = x\text{-values}$$



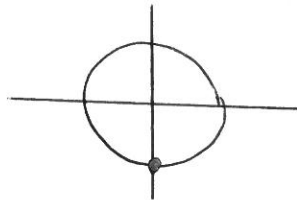
$$\theta = \frac{\pi}{3} : \cos \theta = \frac{1}{2}$$
$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\theta = \pi$$



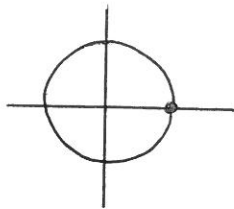
$$\sin \pi = 0 \quad (\text{y-value})$$
$$\cos \pi = -1 \quad (\text{x-value})$$

$$\theta = \frac{3\pi}{2}$$



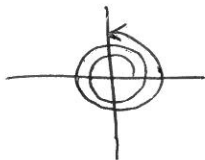
$$\sin \frac{3\pi}{2} = -1$$
$$\cos \frac{3\pi}{2} = 0$$

$$\theta = 2\pi$$



$$\sin 2\pi = 0$$
$$\cos 2\pi = 1$$

$$\theta = \frac{9\pi}{2}$$



$$\sin \frac{9\pi}{2} = 1$$
$$\cos \frac{9\pi}{2} = 0$$