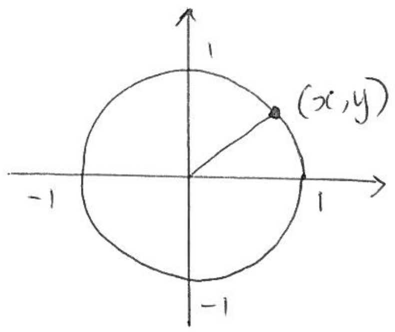
Trig - A new approach

We are now going to think of sin, cos + tan in a different way.

Unit Circle Definition:

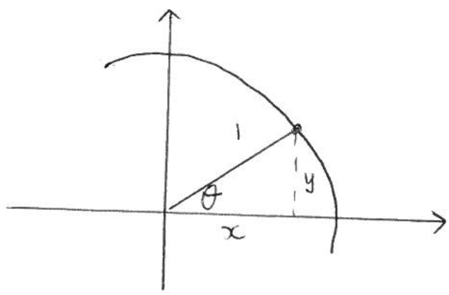


unit circle: $x^2+y^2=1$

We Define:

$\sin\theta = y$ ie: y-value

$\cos\theta = x$ ie: x-value



$\sin\theta = \frac{y}{1}$

$\cos\theta = \frac{x}{1}$

In 1st Quad.
This agrees with our right angle triangle defintion

so $\tan\theta = \frac{y}{x}$

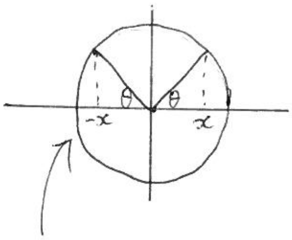
+ $(x, y) = (\cos\theta, \sin\theta)$

And with our new unit circle defintion we can now consider angles bigger than 90°

We will use symmetry to see what happens:

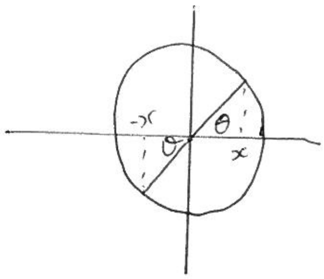
Look at x-value ($\cos\theta$)

2nd Quad:



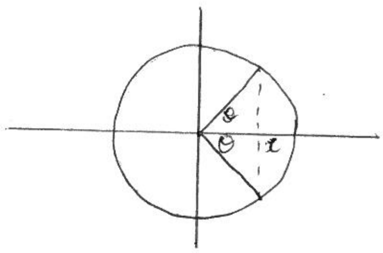
x-value here is a reflection of what happens in 1st quad.
→ Numerical value is same But now its negative

3rd Quad:



Numerical value same But now its neg

4th Quad:



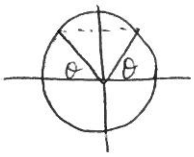
Numerical value same And also its positive

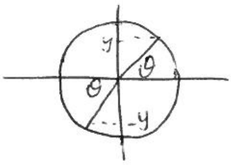
So in other quadrants - Numerical value same but could have different sign.

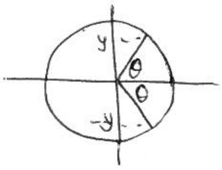
So x-value ie: $\cos\theta$

−	+
−	+

Look at y-values ($\sin\theta$)

2nd Quad  same

3rd Quad  neg

4th Quad  neg

so y-values ie: $\sin\theta$ $\begin{array}{c|c} + & + \\ \hline - & - \end{array}$

Since $\tan\theta = y/x$ we can see $\begin{array}{c|c} - & + \\ \hline + & - \end{array}$

ie: For angles bigger than $90°$ the numerical values of sin, cos + tan repeat but they may be negative.

ie: $\sin\theta$ $\begin{array}{c|c} + & + \\ \hline - & - \end{array}$, $\cos\theta$ $\begin{array}{c|c} - & + \\ \hline - & + \end{array}$, $\tan\theta$ $\begin{array}{c|c} - & + \\ \hline + & - \end{array}$

or summarise these by the ASTC rule

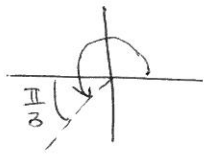
$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$

Now we can find exact values of all angles by putting this info together. We need to know:

- → Exact values
- → Quadrants
- → ASTC rule

eg 13) Find the exact value of $\sin \frac{4\pi}{3}$

- Lets see where this is



3rd Quad

We know this has same numerical value as $\sin \frac{\pi}{3}$ (in 1st quad) ie: $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

But its now neg since sin is neg in 3rd quad.

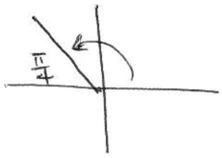
$$\therefore \sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$$

Notice the steps we took:

① Draw

② Quadrant → tells us sign (pos/neg)

③ Angle made with x-axis → tells us exact value eg b) $\cos \frac{3\pi}{4}$

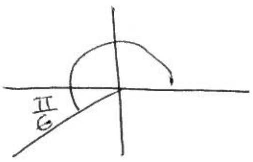


· 2nd quad → cos is neg

· $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\therefore \cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

c) $\tan \frac{7\pi}{6}$

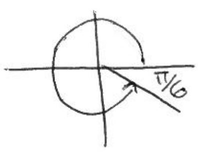


· 3rd quad → tan is pos

· $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\therefore \tan \frac{7\pi}{6} = \frac{1}{\sqrt{3}}$

d) $\sin \frac{11\pi}{6}$

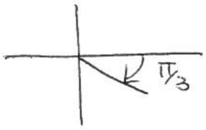


4th quad → sin is neg

$\sin \frac{\pi}{6} = \frac{1}{2}$

$\therefore \sin \frac{11\pi}{6} = -\frac{1}{2}$

e) $\tan\left(-\frac{\pi}{3}\right)$

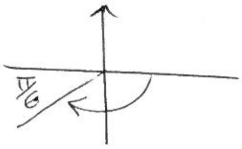


4th quad $\rightarrow$ tan is neg

$\tan\frac{\pi}{3} = \sqrt{3}$

$\therefore \tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}$

f) $\tan\left(-\frac{5\pi}{6}\right)$

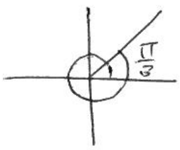


· 3rd quad $\rightarrow$ tan is pos

· $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\therefore \tan\left(-\frac{5\pi}{6}\right) = \frac{1}{\sqrt{3}}$

g) $\cos\frac{7\pi}{3}$



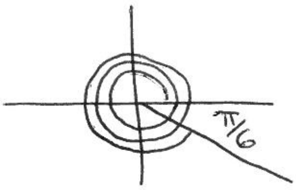
1st quad $\rightarrow$ cos is pos

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\therefore \cos\frac{7\pi}{3} = \frac{1}{2}$

h) $\sin\left(\frac{35\pi}{6}\right)$

$\frac{6\pi}{6} = \pi$



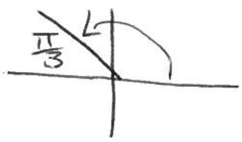
4th quad $\rightarrow$ sin is neg

$\sin\frac{\pi}{6} = \frac{1}{2}$

$\therefore \sin\left(\frac{35\pi}{6}\right) = -\frac{1}{2}$

i) $\sec\left(\frac{2\pi}{3}\right) = \dfrac{1}{\cos\left(\frac{2\pi}{3}\right)}$

and $\cos\frac{2\pi}{3}$:



2nd quad $\rightarrow$ cos is neg

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\therefore \cos\frac{2\pi}{3} = -\frac{1}{2}$

$\therefore \sec\left(\frac{2\pi}{3}\right) = \dfrac{1}{-1/2}$

$= -2$