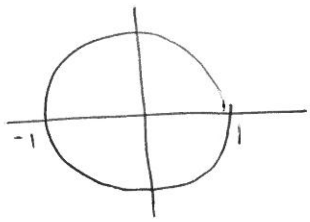
Radians

- another way of measuring angles

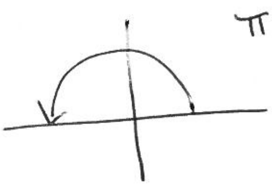
- Circle radius 1

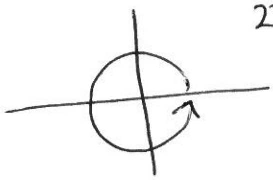


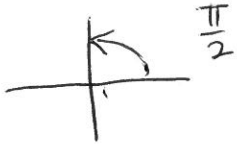
- Imagine wrapping a numberline around the circle
- We can describe the angle by the arc length.

- So the circumference of a circle is $2\pi r$
- circle radius 1 has circumf 2π
- so $360^\circ = 2\pi$

ie: $\boxed{180^\circ = \pi \text{ radians}}$







We see $90^\circ = \frac{\pi}{2}$ radians

$60^\circ = \frac{\pi}{3}$ rads $\left(\frac{1}{3} \text{ of } 180\right)$

$30^\circ = \frac{\pi}{6}$ rads $\left(\frac{1}{6} \text{ of } 180\right)$

$45^\circ = \frac{\pi}{4}$ rads $\left(\frac{1}{4} \text{ of } 180\right)$

Remembering $180^\circ = \pi$ rads we can do any conversion

since $1^\circ = \frac{\pi}{180}$ rads.

so $90^\circ = 90 \times \frac{\pi}{180}$

$= \frac{90\pi}{180}$

$= \frac{\pi}{2}$

Some fractions of 180° are easy to remember

$60^\circ = \frac{\pi}{3}$, $30^\circ = \frac{\pi}{6}$, $45^\circ = \frac{\pi}{4}$, $360^\circ = 2\pi$

Others we can convert

7a) $55^\circ = 55 \times \frac{\pi}{180}$

$= \frac{55\pi}{180}$

$= \frac{11\pi}{36}$

b) $120^\circ = 120 \times \frac{\pi}{180}$

$= \frac{120\pi}{180}$

$= \frac{2\pi}{3}$

(or $120^\circ = 2 \times 60^\circ$
$= \frac{2\pi}{3}$)

c) $23^\circ = 23 \times \frac{\pi}{180}$

$= \frac{23\pi}{180}$

Note: $23^\circ = \frac{23\pi}{180}$ radians

$= 0.401425\ldots$

← we can also write this as a decimal (although we often prefer the exact form).

d) $27^\circ = 27 \times \frac{\pi}{180}$

$= \frac{27\pi}{180}$

$= \frac{3\pi}{20}$

e) $240^\circ = \frac{240\pi}{180}$

$= \frac{4\pi}{3}$

f) $135^\circ = \frac{135\pi}{180}$

$= \frac{3\pi}{4}$ (← note $135^\circ = 3 \times 45^\circ$ ie: $3 \times \frac{\pi}{4}$)

We can also go the other way:

eg 10) Convert the following angles in radians to degrees.

Remember π rads $= 180^\circ$

a) $\frac{\pi}{3} = \frac{180}{3} = 60^\circ$

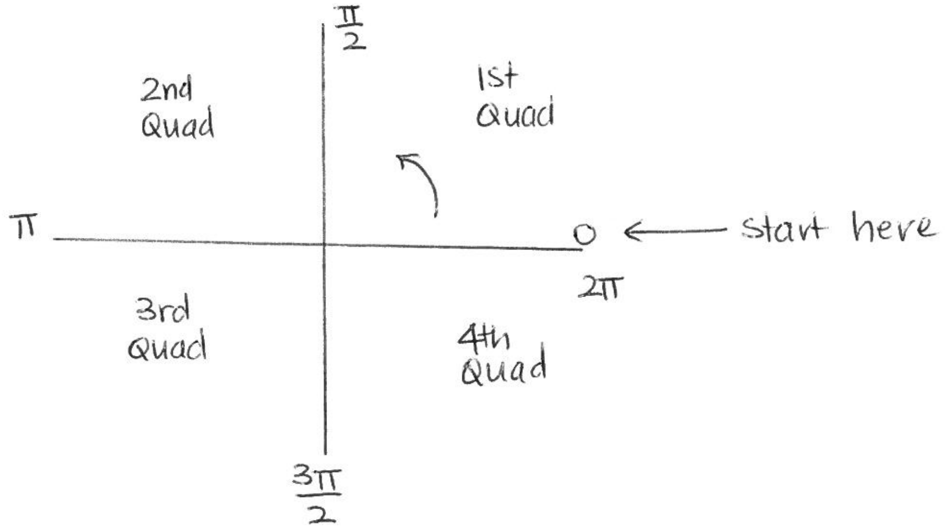
b) $\frac{5\pi}{6} = \frac{5 \times 180}{6} = 150^\circ$

c) $\frac{\pi}{7} = \frac{180}{7} = 25.7^\circ$

d) $\frac{2\pi}{3} = 120^\circ$ ($\leftarrow$ 2 lots of 60°)

Angles on the Plane

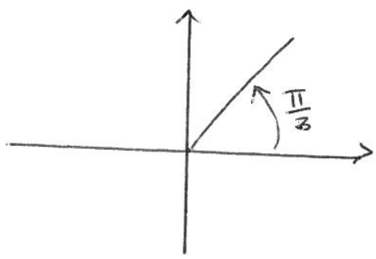
- we can picture our angles as we travel around the plane



- the plane has 4 quadrants
- travel in an anticlockwise direction

Angles on the plane

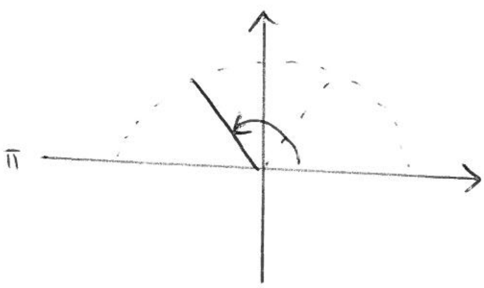
eg 11) a) $\frac{\pi}{3}$



$\frac{1}{3}$ of the way around the top.

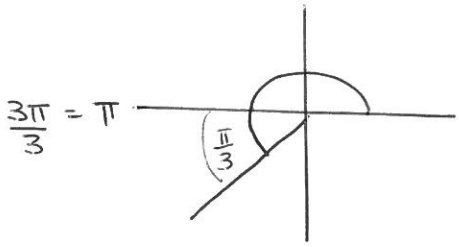
(1st quad)

b) $\frac{2\pi}{3}$



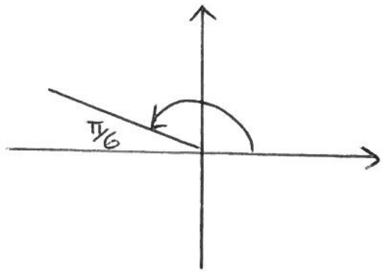
In the 2nd quad.

c) $\frac{4\pi}{3}$



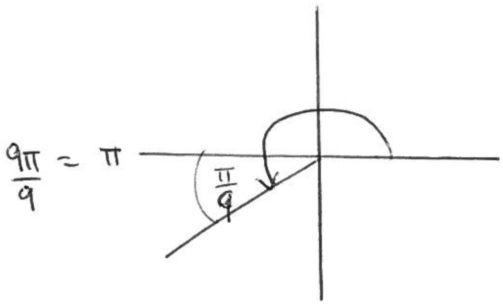
3rd quad

d) $\frac{5\pi}{6}$



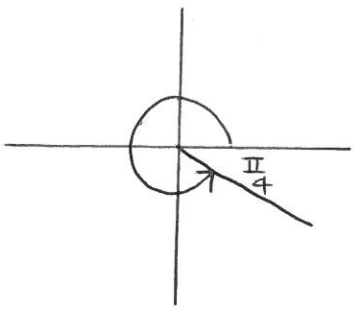
2nd quad

e) $\frac{10\pi}{9}$



3rd quad

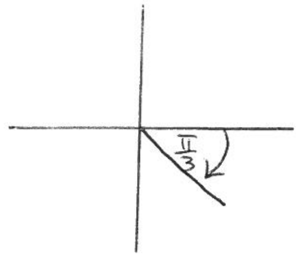
f) $\frac{7\pi}{4}$



4th quad

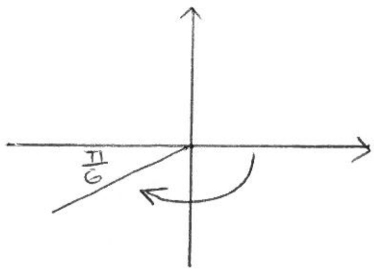
g) $-\frac{\pi}{3}$

neg = change direction
ie: go backwards



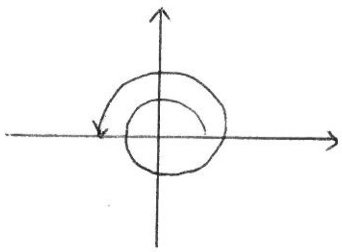
(4th quad)

h) $-\frac{5\pi}{6}$



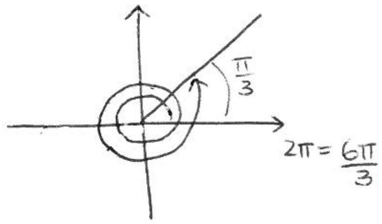
3rd quad

i) 3π



on the
axis.

j) $\frac{13\pi}{3}$

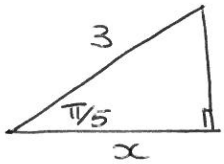


so $\frac{13\pi}{3}$ is in
the same position
as $\frac{\pi}{3}$

Back to triangles

From now we will use radians when working with trig ratios

eg:



$$\cos\frac{\pi}{5} = \frac{x}{3}$$

$$\therefore\ x = 3\cos\frac{\pi}{5}$$

↑ now change calc to radians

$$= 3 \times 0.809\ldots$$

$$= 2.427$$

All trig ratios can be found on calc when we need them

But some angles occur so frequently and are important that we want to get to know their ratios exactly.

These angles are 30°, 45°, 60°

ie: $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$

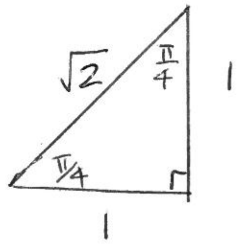
Certain right angled triangles give us these exact values.

Standard Triangles

- The first triangle = right angled isoceles : sides = 1

  angles = $45^\circ = \frac{\pi}{4}$

  hyp = $c^2 = 1^2 + 1^2 = 2$

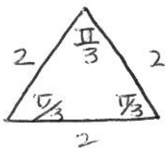


$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

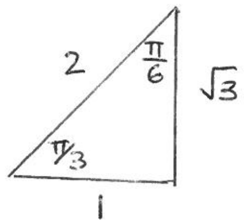
$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\tan\frac{\pi}{4} = 1$$

- The second triangle = equilateral triangle : sides = 2



Split in half to create right angled triangle



$\leftarrow$

$$x^2 + 1^2 = 2^2$$
$$x^2 = 3$$
$$x = \sqrt{3}$$

So

$$\sin\frac{\pi}{6} = \frac{1}{2} \qquad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \qquad \tan\frac{\pi}{3} = \sqrt{3}$$

We need to remember these exact values!

	sin	cos	tan
$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$

Notice the pattern in the table

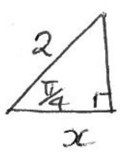
or Remember the triangles.

- These are called exact values or exact ratios

Eg 12) Find the exact value of $\tan \frac{5\pi}{15}$

$$\tan \frac{5\pi}{15} = \tan \frac{\pi}{3} = \sqrt{3}$$

eg 13)



Find x

$$\cos \frac{\pi}{4} = \frac{x}{2}$$

$$x = 2\cos\frac{\pi}{4}$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{2}{\sqrt{2}}$$