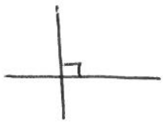
Intro to Trigonometry

Some basic geometry:

Angles = amount of turning as we travel round a circle

Full circle = 360°
(1 revolution)



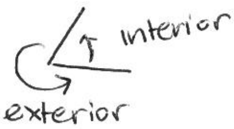
perpendicular lines meet at right angles = 90°

180° - straight angle

obtuse angle - between 90° and 180°

acute angle - between 0° and 90°

Some conventions:

- 

  convention - we give the interior angle

- 

  we travel in an anticlockwise way when measuring angles

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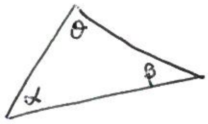
  we usually use greek letters to denote an angle $\theta, \alpha, \beta, \phi$,

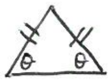
  Here θ = interior angle
  $360^\circ - \theta$ = exterior angle

Triangles

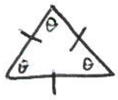


The sum of angles in a triangle is $180°$

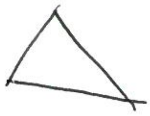
ie: $\alpha + \beta + \theta = 180°$



isoceles



equilateral
$\theta = 60°$

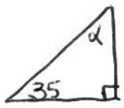


scalene

hypotenuse

right-angled triangle

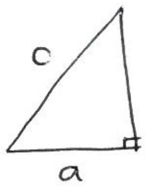
eg:



so $\alpha = 180 - 90 - 35 = 55°$

Pythagoras

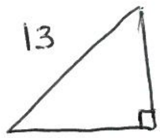
All right angled triangles obey Pythagoras Theorem



$$\boxed{a^2 + b^2 = c^2}$$

" The square of the hypotenuse equals the sum of the squares of the other 2 sides "

eg:

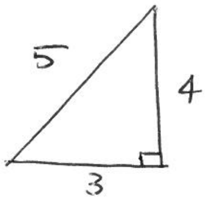


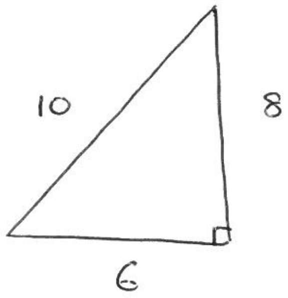
$$a^2 + 12^2 = 13^2$$
$$a^2 = 13^2 - 12^2$$
$$a^2 = 25$$
$$a = 5$$

(Try Q1 and 2 Prob set)

Similar Triangles

- have the same shape

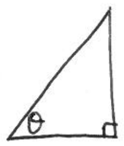




- their sides are in proportion

$\Rightarrow$ their corresponding angles are the same

- Since triangles formed by certain angles are proportional, we describe their angles by looking at the ratio of sides.

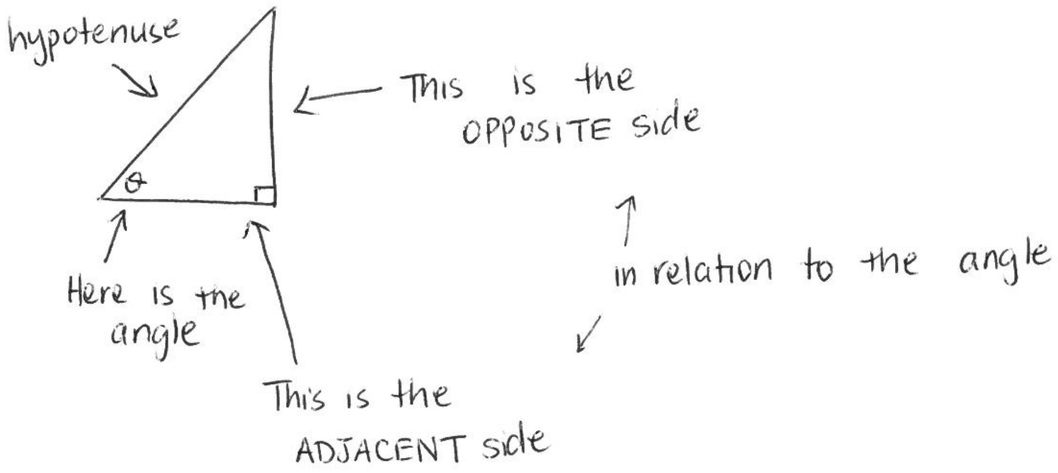


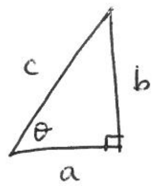
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \text{(sine)}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \text{(cosine)}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \quad \text{(tangent)}$$

ie :

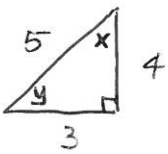




$$\sin\theta = \frac{b}{c}$$

$$\cos\theta = \frac{a}{c}$$

$$\tan\theta = \frac{b}{a}$$



$$\sin x = \frac{3}{5} \qquad \sin y = \frac{4}{5}$$

$$\cos x = \frac{4}{5} \qquad \cos y = \frac{3}{5}$$

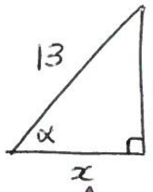
$$\tan x = \frac{3}{4} \qquad \tan y = \frac{4}{3}$$

Notice : $\boxed{\tan\theta = \dfrac{\sin\theta}{\cos\theta}}$

check
$$\frac{\sin\theta}{\cos\theta} = \frac{b/c}{a/c}$$
$$= \frac{b}{c} \times \frac{c}{a}$$
$$= \frac{b}{a}$$
$$= \tan\theta$$

Eg3) If $\sin\alpha = \frac{5}{13}$, sketch a right angled triangle to represent this and find $\cos\alpha$ and $\tan\alpha$.

$\sin\alpha = \frac{5}{13}$



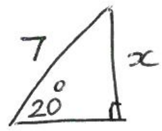
$x^2 + 5^2 = 13^2$

$x^2 = 169 - 25$

$x = 12$

$\therefore \cos\alpha = \frac{12}{13}$

$\tan\alpha = \frac{5}{12}$

eg4) Given the triangle 

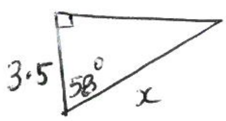
, Find x.

We have opposite + hypotenuse → use sine

$\therefore \sin 20^\circ = \frac{x}{7}$

$\therefore x = 7\sin 20^\circ$

$= 2.39$

eg5) 

Find x

$\cos 58^\circ = \frac{3.5}{x}$

$\therefore x = \frac{3.5}{\cos 58^\circ}$

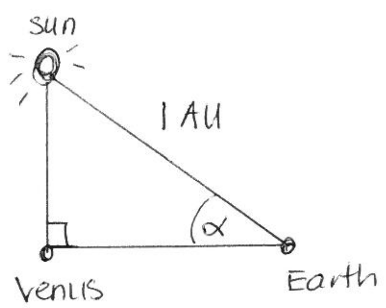
$= 6.605$

Try Q6 Probset

Eg)

7. The elongation α of a planet is the angle formed by the planet, earth and sun. When Venus achieves its maximum elongation of $46.3°$, the earth, Venus and the sun form a triangle with a right angle at Venus. Find the distance between Venus and the Earth in astronomical units. (By definition the distance between the earth and the sun is 1 AU).

Picture whats happenning:



$\alpha = 46.3°$

We want distance from venus to earth ← call it x.

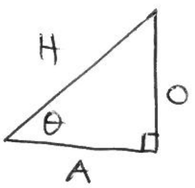
$$\therefore \cos 46.3 = \frac{x}{1}$$

$$\therefore x = \cos 46.3$$
$$= 0.69$$

$\therefore$ dist is 0.69 AU.

Trig Ratios

Notice there are actually 6 different ratios in a right angled triangle



$$\sin\theta = \frac{O}{H}$$

$$\cos\theta = \frac{A}{H}$$

$$\tan\theta = \frac{O}{A}$$

Also

$$\sec\theta = \frac{H}{A}$$ (secant)

$$\operatorname{cosec}\theta = \frac{H}{O}$$ (co-secant)

$$\cot\theta = \frac{A}{O}$$ (co-tangent)

Note:

$$\sec\theta = \frac{1}{\cos\theta}$$

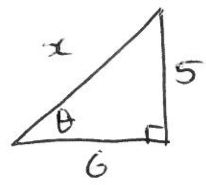
$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

- everything can be written in terms of sin + cos.

eg 8) Given $\tan\theta = \frac{5}{6}$, sketch a right angled triangle to represent this, and find the other 5 trig ratios.

$\tan\theta = \frac{5}{6}$



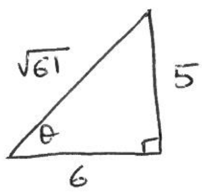
$\therefore x^2 = 5^2 + 6^2$

$= 25 + 36$

$x^2 = 61$

$x = \sqrt{61}$

$\therefore$ we have



$\sin\theta = \frac{5}{\sqrt{61}}$ $\qquad$ $\operatorname{cosec}\theta = \frac{\sqrt{61}}{5}$

$\cos\theta = \frac{6}{\sqrt{61}}$ $\qquad$ $\sec\theta = \frac{\sqrt{61}}{6}$

$\cot\theta = \frac{6}{5}$