## Sum of an AP

Eg: Look at the sum of the first 100 positive integers:

ie: $1+2+3+\cdots+100$

Notice we can pair up numbers

$$1+2+3+\cdots+98+99+100$$

$$= \underbrace{101+101+101+\cdots+101}_{50 \text{ times}}$$

$$= 50\,(1+100)$$

$\frac{n}{2}$ first last

Let $\ell$ = last term $\therefore S_n = \frac{n}{2}(a+\ell)$

In general: For any AP

$$S_n = a + (a+d) + (a+2d) + \cdots + (\ell-2d) + (\ell-d) + \ell$$

$$= \frac{n}{2}(a+\ell)$$

Also since $\ell = a+(n-1)d$ ← $n^{th}$ term

$$\text{sum} = \frac{n}{2}(a+\ell) = \frac{n}{2}(a+a+(n-1)d)$$

$$= \frac{n}{2}(2a+(n-1)d)$$

$$\boxed{\therefore \text{Sum of first } n \text{ terms of AP} = S_n = \frac{n}{2}(a+\ell) = \frac{n}{2}(2a+(n-1)d)}$$

Egs 6)

a) $6, 10, 14, \dots$ Find $S_{20}$.

Here $a = 6$, $d = 4$, $n = 20$

$$\therefore S_{20} = \frac{n}{2}(2a + (n-1)d)$$
$$= \frac{20}{2}(2(6) + 19(4))$$
$$= 10(12 + 76)$$
$$= 880$$

b) Find $13 + 16 + \dots + 109$

Here $a = 13$, $d = 3$, $l = 109$ $\therefore$ Use $S_n = \frac{n}{2}(a + l)$

But we need $n$.

First find $n$: want $n$ for $a_n = 109$

ie: $a + (n-1)d = 109$

ie: $13 + (n-1)3 = 109$

$13 + 3n - 3 = 109$

$10 + 3n = 109$

$3n = 99$

$n = 33$

$\therefore$ we want $S_{33} = \frac{33}{2}(13 + 109)$

$= 2013$

c) $7 + 12 + 17 + \cdots$ Find a formula for $S_n$.

$a = 7$, $d = 5$ $\qquad S_n = \frac{n}{2}(2a + (n-1)d)$

∴ We want a formula for the sum in terms of $n$.

$$S_n = \frac{n}{2}(2(7) + (n-1)5)$$
$$= \frac{n}{2}(14 + 5n - 5)$$
$$= \frac{n}{2}(9 + 5n)$$

d) $9 + 5 + 1 + \cdots$ Find $S_{100}$

so $a = 9$, $d = -4$, $n = 100$

$$\therefore S_{100} = \frac{100}{2}(2(9) + (100-1)(-4))$$
$$= 50(18 + 99(-4))$$
$$= 50(-378)$$
$$= -18900$$

# Geometric Progressions

eg $3, 6, 12, 24, \ldots$

$3 \xrightarrow{\times 2} 6 \xrightarrow{\times 2} 12$

- multiplying by a constant amount
- notice the ratio of terms is constant

ie: $\frac{6}{3} = \frac{12}{6} = \frac{24}{12} = 2$

We say common ratio $= r = 2$

test for GP : $r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots \frac{a_n}{a_{n-1}}$

Eg Find the first term a + common ratio r.

a) $16, 64, 256, \ldots$

$a = 16$

common ratio $\frac{64}{16} = \frac{256}{64} = 4 \quad \therefore r = 4$

b) $8, 4, 2, 1, \ldots$

$a = 8, \quad r = \frac{1}{2}$

c) $-1, 1, -1, 1, \ldots$

$a = -1, \quad r = -1$

d) $2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \ldots$

$a = 2, \quad r = \frac{1}{3}$ (check $\frac{2/3}{2} = \frac{1}{3}$ ; $\frac{2/9}{2/3} = \frac{2}{9} \times \frac{3}{2} = \frac{3}{9} = \frac{1}{3}$)

# $n^{th}$ term of Geometric Progression

eg: 3, 6, 12, 24, ...

| term nº (n) | term value $a_n$ | |
|---|---|---|
| 1 | 3 | $\leftarrow 3$ |
| 2 | 6 | $\leftarrow 3 \times 2 = 3 \times (2)^1$ |
| 3 | 12 | $\leftarrow 3 \times 2 \times 2 = 3 \times (2)^2$ |
| 4 | 24 | $\leftarrow 3 \times (2)^3$ |

$$\underset{a}{3} \times (\underset{r}{2})^{\overset{n-1}{3}}$$

ie: $\boxed{a_n = ar^{n-1}}$

In general GP: $a, ar, ar^2, ar^3, \ldots$

$n^{th}$ term $= a \times r$ mult by itself $n-1$ times

ie: $a_n = ar^{n-1}$

Eg Find formula for $n^{th}$ term + use it to calc $100^{th}$ term

a) 16, 64, 256, ...

$a = 16$, $r = 4$

$\therefore a_n = ar^{n-1} = 16(4)^{n-1}$

$\therefore a_{10} = 16(4)^9 = 4\,194\,304$

b) $8, 4, 2, 1, \ldots$

$a = 8, \quad r = \frac{1}{2}$

$\therefore a_n = 8\left(\frac{1}{2}\right)^{n-1} = \frac{8}{2^{n-1}} = \frac{2^3}{2^{n-1}} = 2^{3-(n-1)} = 2^{4-n}$

So $a_n = 2^{4-n}$

$\therefore a_{10} = 2^{4-10} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$

c) $-1, 1, -1, 1, \ldots$

$a = -1, \quad r = -1$

$\therefore a_n = (-1)(-1)^{n-1}$

$= (-1)^n$

So $a_{10} = (-1)^{10} = 1$

d) $2, \frac{2}{3}, \frac{2}{9}, \ldots$

$a = 2, \quad r = \frac{1}{3}$

$a_n = 2\left(\frac{1}{3}\right)^{n-1} = \frac{2}{3^{n-1}}$

$\therefore a_{10} = \frac{2}{3^9}$

Eg The third term of a GP is 20 + the 6th term is 160.
Find a) the first 3 terms
b) the formula for the nth term
c) the 100th term.

- We're told $a_3 = 20$, $a_6 = 160$
- It's a GP so $ar^2 = 20$ ① , $ar^5 = 160$ ② Solve to find a + r

②÷① : $r^3 = \frac{160}{20} = 8$

$\therefore r = 2$

sub back in ① : $a(2)^2 = 20$

$a = 5$

$\therefore a = 5, r = 2$

So first 3 terms are 5, 10, 20,

b) $n^{th}$ term : $a_n = ar^{n-1}$

$= 5(2)^{n-1}$

c) $100^{th}$ term $= a_{100} = 5(2)^{99}$

Eg) The third term of a GP is 64 + the 8th term is 2. Find the 17th term

$a_3 = 64 \rightarrow ar^2 = 64$ —— ①

$a_8 = 2 \rightarrow ar^7 = 2$ —— ②

② ÷ ① $\quad r^5 = \frac{2}{64}$

$r^5 = \frac{1}{32}$

$r = \frac{1}{2}$

sub into ① : $a\left(\frac{1}{2}\right)^2 = 64$

$\frac{a}{4} = 64$

$a = 256$

So $\quad a = 256, \ r = \frac{1}{2}$

Want 17th term ie: $a_{17} = ar^{16}$

$= 256\left(\frac{1}{2}\right)^{16}$

$= \frac{256}{2^{16}}$

$= \frac{1}{256}$