Arithmetic + Geometric Progressions

- Progression / Sequence = ordered list of numbers.

eg 1) 3, 5, 7, 9, ...

2) 17, 12, 7, 2, ...

3) 3, 6, 12, 24, ...

- each number in sequence is called a term

3 , 5 , 7 , ...

↑ ↑

1st term 2nd term ...

a_1 a_2

- a_n = n^{th} term
- We give the first term a special name, a.

- Some sequences have patterns (so we can predict things)
- we're going to look at 2 types of patterns

1. Arithmetic Progression - adds on a constant amount

3, 5, 7, 9, ... adding 2

↷ ↷

+2 +2

2. Geometric Progression - multiplies by a constant amount

eg: 3, 6, 12, 24 mult by 2.

↷ ↷

×2 ×2

Arithmetic Progressions

eg 1) $3, 5, 7, 9, \cdots$

$\overset{+2}{\curvearrowright}\ \overset{+2}{\curvearrowright}\ \cdots$

- We are adding 2
- Notice the difference between terms is constant

ie: $5-3 = 7-5 = 9-7 = \cdots 2$

We say common difference $= d = 2$

eg 2) $17, 12, 7, 2, \cdots$

$\overset{-5}{\curvearrowright}\ \overset{-5}{\curvearrowright}\ \overset{-5}{\curvearrowright}$

- Subtracting 5 to get next term
- or adding -5 to get next term

$\therefore$ Here $d = -5$ (check $12-17 = 7-12 = -5$)

eg 3) $3, 6, 12, 24, \cdots$ is not arithmetic

since we're not adding by const amt

ie: $6-3 \neq 12-6 \cdots$

test for AP: $d = a_2 - a_1 = a_3 - a_2 = \cdots = a_n - a_{n-1}$

Eg 1) For each of the following APs, write down the first term a and common difference d.

a) $7, 11, 15, 19, \cdots$

$a = 7, \quad d = 4$

b) $10, 4, -2, -8, \cdots$

$a = 10$

$d = 4 - 10 = -2 - 4 = -6$

c) $-3, -6, -9, -12, \cdots$

$a = -3$

$d = -3$

d) $1, \frac{3}{2}, 2, \frac{5}{2}, \cdots$

$a = 1$

$d = \frac{1}{2}$

Describing sequences

3, 5, 7, ...

↑ dots mean this pattern continues forever

But better to describe this with formula for n^{th} term

3, 5, 7, ...
↑ ↑ ↑
a_1 a_2 a_3 ...

Here $a_n = 1 + 2n$

so $a_4 = 1 + 2(4) = 9$

$a_5 = 1 + 2(5) = 11$

Eg: Using the given n^{th} term formula, write the first 3 terms of the sequence described.

a) $a_n = 3n - 1$: 2, 5, 8, 11, ... ← AP (d=3)

↑ ↑ ↑ ↑
n=1 n=2 n=3 n=4
3-1 6-1 9-1

b) $a_n = 3n$: 3, 6, 9, 12, ... ← AP (d=3)

c) $a_n = n^2$: 1, 4, 9, 16, ... ← neither

d) $a_n = -5 - 4n$: -9, -13, -17, ... ← AP (d=-4)

↑ ↑ ↑
n=1 n=2 n=3
-5-4(2) -5-4(3)

nth term formula

eg: 3, 5, 7, ...

a_1 a_2 a_3

- since this has a pattern we can start to predict terms

$a_4 = 9$

$a_5 = 11$

⋮

So :

n (term number)	a_n = value of n^{th} term			
1	3			
2	5	←	3+2	= 3 + (1)2
3	7	←	3+2+2	= 3 + (2)2
4	9	←		3 + (3)2
5	11	←		3 + (4)2
⋮	⋮			
n	3 + (n−1)2			

1 less than term number

So $a_n = 3 + (n-1)2$

a d

$$\therefore \boxed{a_n = a + (n-1)d}$$

In general: AP : $a, a+d, a+2d, \ldots$

$\therefore n^{th}$ term = add d a total of $n-1$ times to first term a

ie: $a_n = a + (n-1)d$

Eg 3) For each of the following APs, find the n^{th} term + use this to calculate the 100th term.

a) $8, 14, 20, \cdots$

$a = 8, \quad d = 6$

$\therefore a_n = a + (n-1)d = 8 + (n-1)6$

$= 8 + 6n - 6$

$= 2 + 6n$

So $a_n = 2 + 6n$

$\therefore$ 100th term $= a_{100} = 2 + 6(100) = 602$.

b) $53, 49, 45, \cdots$

$a = 53 \quad d = -4$

$\therefore a_n = 53 + (n-1)(-4)$

$= 53 - 4n + 4$

$= 57 - 4n$

So $a_{100} = 57 - 4(100)$

$= 57 - 400$

$= -343$

4) The first term of an AP is 7 and the tenth term is -20. Find the n^{th} term.

$a = 7$

$a_{10} = -20$

We want $a_n = a + (n-1)d$

(Aim: Find a and d)

$a = 7$

$a_{10} = a + (10-1)d$

$= a + 9d = -20$

ie: $7 + 9d = -20$

$9d = -27$

$d = -3$

$\therefore a_n = 7 + (n-1)(-3)$

$= 7 - 3n + 3$

$= 10 - 3n$

5) The third term of an AP is 20 + sixth term is 47. Find the sequence + 20th term.

So $a_3 = a + 2d = 20$ —①

$a_6 = a + 5d = 47$ —②

② - ①: $3d = 27$

$\therefore d = 9$

sub back in ①: $a + 2(9) = 20$

$a = 2$

$\therefore$ Seq: 2, 11, 20, ... + $a_n = 2 + (n-1)9$

$\therefore a_{20} = 2 + 19 \times 9 = 173$