7a) Solve $\frac{x+3}{x-4} \leq 2$

Case Method: (note $x \neq 4$)

Case1: Denom pos

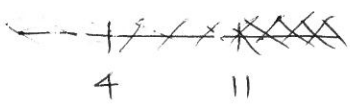
$x-4>0$

$x>4$

$x+3 \leq 2(x-4)$

$x+3 \leq 2x-8$

$11 \leq x$



$x \geq 11$

Case 2: Denom neg

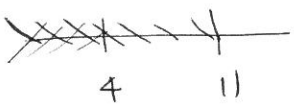
$x-4<0$

$x<4$

$x+3 \geq 2(x-4)$

$x+3 \geq 2x-8$

$11 \geq x$



$x < 4$ (note $x \neq 4$)

$\therefore$ Soln: $x<4, x \geq 11$

Other Methods of Solving

Method 2 : $\dfrac{x+3}{x-4} \leq 2$

Mult both sides by denominator squared.

$$\frac{x+3}{x-4} \leq 2 \quad \times (x-4)^2$$

$$(x+3)(x-4) \leq 2(x-4)^2$$

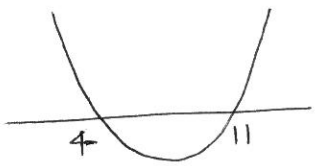
ie: $x^2+3x-4x-12 \leq 2(x^2-8x+16)$

$$x^2-x-12 \leq 2x^2-16x+32$$

$$0 \leq x^2-15x+44$$

ie: $x^2-15x+44 \geq 0$

ie: $(x-4)(x-11) \geq 0$



$x < 4$, $x \geq 11$

Method 3 : $\dfrac{x+3}{x-4} \leq 2$

Bring everything on one side + test signs :

$$\frac{x+3}{x-4} - 2 \leq 0$$

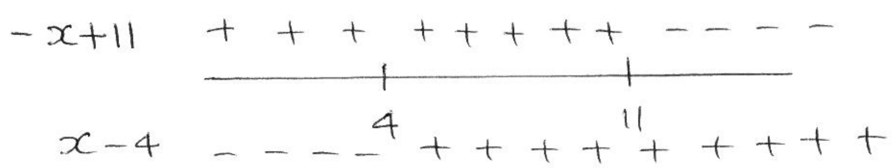
ie: $\dfrac{x+3-2(x-4)}{x-4} \leq 0$

ie: $\dfrac{x+3-2x+8}{x-4} \leq 0$

ie: $\dfrac{-x+11}{x-4} \leq 0$

↖ want this to be neg

∴ look at sign of top + bottom



Choose area where top + bottom have different signs

ie: $x < 4$, $x \geqslant 11$

eg b) Solve $\dfrac{x^2+8x+12}{x+1} > 0$

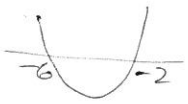
ie: $\dfrac{(x+6)(x+2)}{x+1} > 0$

Test signs

$(x+6)(x+2)$ + + + + − − − − − + + + + + + + +

$x+1$ − − − − − − − − − + + + + +

−6 −2 −1



want ineq to be pos

∴ Need same signs on top + bottom

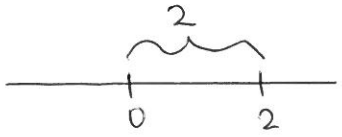
∴ $-6 < x < -2$, $x > -1$

Absolute Value Inequalities

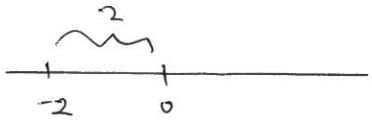
Recall Absolute Value :

$|x|$ = distance of x from 0

ie: $|2| = 2$



$|-2| = 2$



Defn : $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Think of $|x| = |x - 0|$ = dist of x from 0

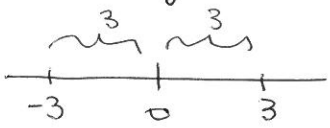
$|x - 1|$ = dist of x from 1

Absolute Value Equations

eg8a) Solve $|x| = 3$

↖ dist of x from 0

ie:



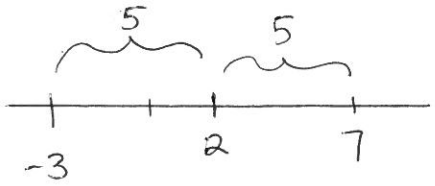
$\therefore x = \pm 3$

Infact $\boxed{\text{If } |x| = a \Rightarrow x = \pm a}$

b) Solve $|x-2| = 5$

↑ dist of x from 2 is 5

ie:



so $x = -3, 7$

Algebraically: $|x-2| = 5$

then $x-2 = 5$ or $x-2 = -5$

$\therefore x = 7$ $x = -3$

[check: $x=7$: LHS $= |7-2| = 5$ ✓
$x=-3$: $|-3-2| = |-5| = 5$ ✓]

c) Solve $3|x+5| + 6 = 15$

$\therefore 3|x+5| = 9$

$|x+5| = 3$

$\therefore x+5 = 3$ $x+5 = -3$

$x = -2$ $x = -8$

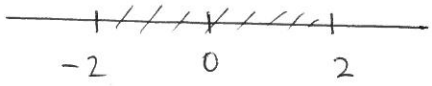
[Check $x=-2$: $3|-2+5| + 6 = 15$ ✓
$x=-8$: $3|-8+5| + 6 = 15$ ✓]

Absolute Value Inequalities

$|x| < 2$

↑

dist of x from 0 is less than 2



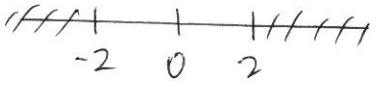
ie: $-2 < x < 2$

Rule: $\boxed{|x| < a \Rightarrow -a < x < a}$

$|x| > 2$

↑ ↖

dist of x from 0 is greater than 2



ie: $x < -2$, $x > 2$

Rule: $\boxed{|x| > a \Rightarrow x < -a, \; x > a}$

Eg 9a) Solve $|3x| < 24$

$\therefore -24 < 3x < 24$

ie: $-8 < x < 8$

b) $|x-5| < 3$

$\therefore -3 < x-5 < 3$

$\therefore 2 < x < 8$

c) $\left|\frac{3x-1}{2}\right| \leq 10$

$\therefore -10 \leq \frac{3x-1}{2} \leq 10$

$\therefore -20 \leq 3x-1 \leq 20$

$-19 \leq 3x \leq 21$

$-\frac{19}{3} \leq x \leq 7$

d) $\left|\frac{x-2}{3}\right| > 2$

So $\frac{x-2}{3} < -2$ or $\frac{x-2}{3} > 2$

$\therefore < -6$ $\quad x-2 > 6$

$x < -4$ $\quad x > 8$

e) $8 - |2x-1| \leq 6$

$-|2x-1| \leq -2$

$|2x-1| \geq 2$

$\therefore 2x-1 \leq -2$ or $2x-1 \geq 2$

$2x \leq -1$ $\quad$ $2x \geq 3$

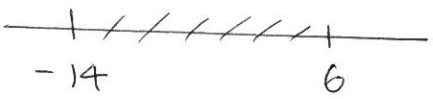
$x \leq -\frac{1}{2}$ $\quad$ $x \geq \frac{3}{2}$

f) $2 < |x+4| < 10$

This says $|x+4| < 10$ AND $|x+4| > 2$

ie: $-10 < x+4 < 10$ $\quad$ $x+4 < -2$, $x+4 > 2$

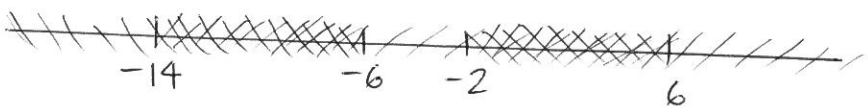
$-14 < x < 6$ $\quad$ $x < -6$ $\quad$ $x > -2$



AND

Want both to hold :

ie:



$\therefore$ Soln : $-14 < x < 6$, $-2 < x < 6$