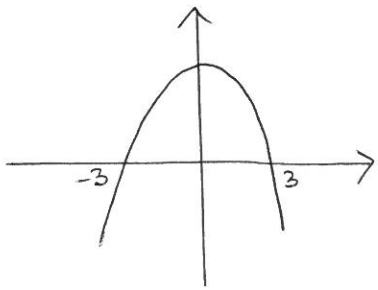
e) Solve $9 - x^2 \geqslant 0$

Sketch $y = 9 - x^2$

$= (3 + x)(3 - x)$



$\therefore \quad -3 \leq x \leq 3$

f) Solve $4x^2 + 3x < 2x^2 + 2$

ie: $2x^2 + 3x - 2 < 0$

Look at parabola $y = 2x^2 + 3x - 2$

$= 2x^2 + 4x - x - 2$

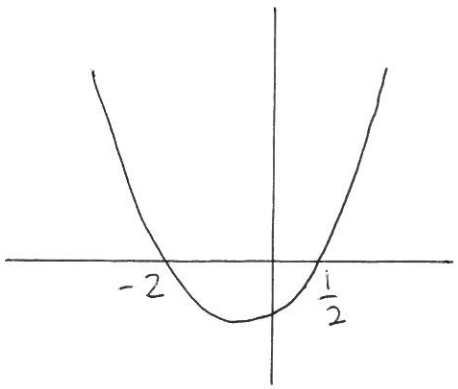
$= 2x(x+2) - (x+2)$

$= (x+2)(2x-1)$

$P = -4$, $S = 3$ } $4, -1$

x int $\rightarrow y = 0$

$x = -2, \frac{1}{2}$



For $y < 0$ take part of parabola below axis

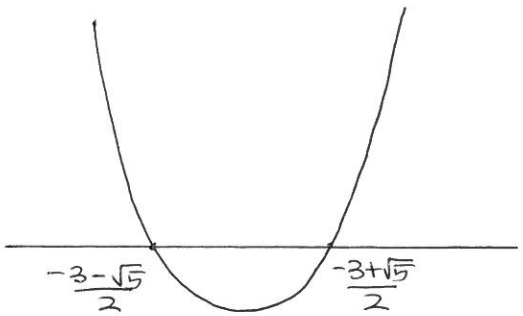
ie: $-2 < x < \frac{1}{2}$

g) Solve $x^2+3x+1 \geqslant 0$

Look at $y=x^2+3x+1$

↖ can't factorise but can find solutions $x = \frac{-3 \pm \sqrt{9-4}}{2}$

$= \frac{-3 \pm \sqrt{5}}{2}$

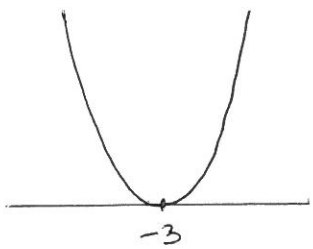


$\therefore \quad x \leq \frac{-3-\sqrt{5}}{2}, \quad x \geqslant \frac{-3+\sqrt{5}}{2}$

h) Solve $x^2+6x+9 > 0$

$\therefore$ Look at $y=x^2+6x+9$

$= (x+3)^2$



This is always pos except when $x=-3$

$\therefore x<-3, \quad x>-3$

(But $x \neq -3$)

1) Solve $x^2+3x+3 > 0$

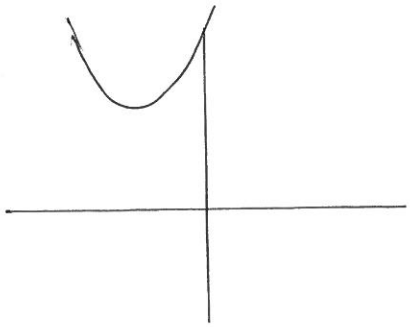
Look at $y = x^2+3x+3$

↖ can't factorise

Notice $\Delta = b^2-4ac = 3^2-4(3) < 0$

∴ No solutions

∴ No x intercepts.



· Parabola exists - it doesn't cut x-axis

· Since its facing up it must be above x axis

∴ It is always giving positive values.

∴ Soln : $x \in \mathbb{R}$

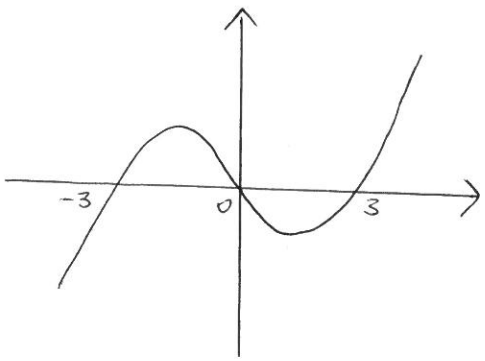
Polynomial Inequalities

eg 5a) Solve $x^3 - 9x > 0$

Look at $y = x^3 - 9x$

$= x(x^2 - 9)$

$= x(x+3)(x-3)$

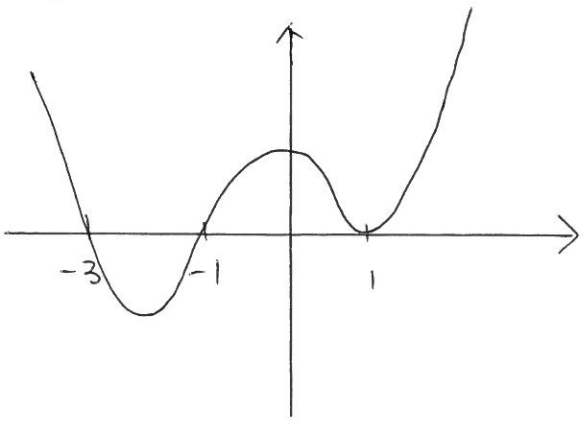


we want y-values to be positive

ie: $-3 < x < 0$, $x > 3$

b) Solve $(x+3)(x+1)(x-1)^2 \leq 0$

Sketch $y = (x+3)(x+1)(x-1)^2$ ← . deg 4
. double root at $x=1$
. facing up

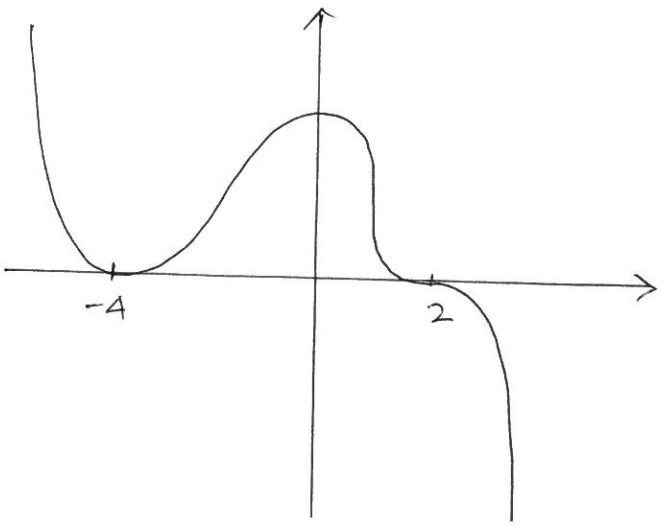


want $y \leq 0$

This corresponds to $-3 \leq x \leq -1$ or $x = 1$

c) Solve $(2-x)^3(4+x)^2 \leq 0$

First sketch $y=(2-x)^3(4+x)^2$



- deg = 5
- $x = 2$ ← mult 3 ~
  $x = -4$ ← mult 2 ∪
- test $x = -5$: $y = (7^3)(-1)^2$
  > 0

$\therefore \quad x \geq 2 \quad , \quad x = -4$

xs in the Denominator

eg6a) Solve $\dfrac{1}{x-2} < 3$

we don't know if the denominator is positive or negative → Lets take cases : (Note $x \neq 2$)

Case 1 : Denom pos

$x-2 > 0$ (ie: $x > 2$)

$1 < 3(x-2)$

$1 < 3x-6$

$7 < 3x$

$\frac{7}{3} < x$

ie: $x > \frac{7}{3}$

Case 2 : Denom neg

$x-2 < 0$ (ie: $x > 2$)

$1 > 3(x-2)$

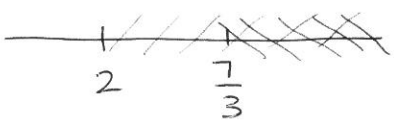
$1 > 3x-6$

$7 > 3x$

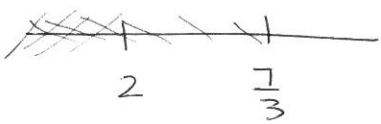
$\frac{7}{3} > x$

ie: $x < \frac{7}{3}$

ie:



$\therefore x > \frac{7}{3}$



$\therefore x < 2$

$\therefore$ Soln : $x > \frac{7}{3}, x < 2$

eg b) Solve $\frac{12}{3x+2} > 4$

(Note : $x \neq -\frac{2}{3}$)

Using case method :

Case 1 : Denom pos :

$3x+2>0$

$x > -\frac{2}{3}$

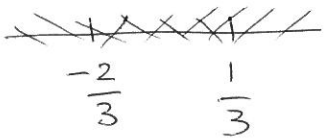
$\therefore 12 > 4(3x+2)$

$12 > 12x+8$

$4 > 12x$

$\frac{4}{12} > x$

$\frac{1}{3} > x$



$\therefore -\frac{2}{3} < x < \frac{1}{3}$

Case 2 : Denom neg

$3x+2<0$

$x < -\frac{2}{3}$

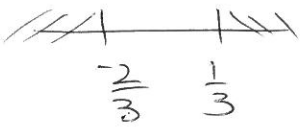
$12 < 4(3x+2)$

$12 < 12x+8$

$4 < 12x$

$\frac{4}{12} < x$

$\frac{1}{3} < x$



No soln

$\therefore$ Soln : $-\frac{2}{3} < x < \frac{1}{3}$