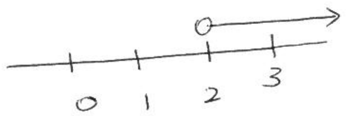
Inequalities

= expression involving $>$ $\geq$ $<$ $\leq$ signs

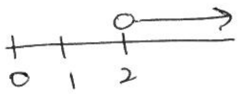
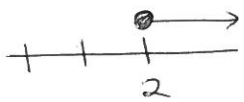
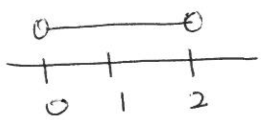
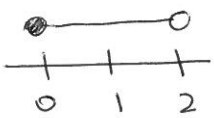
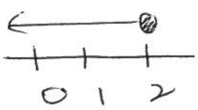
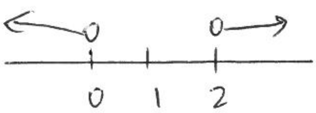
eg: $5>3$ "5 is greater than 3"

$3<5$ "3 is less than 5"

$x>2$ ← x is a number bigger than 2



- we can picture these on a number line

expression	n° line	interval
$x>2$		$(2,\infty)$
$x\geq 2$		$[2,\infty)$
$0<x<2$		$(0,2)$
$0\leq x<2$		$[0,2)$
$x\leq 2$		$(-\infty,2]$
$x<0,\ x>2$		$(-\infty,0)\cup(2,\infty)$

↑ union

Eg: Solve $x+1 \geqslant 7$

Notice some values of x satisfy this inequality + some don't.

$x=1$	:	$2 \geqslant 7$	No
$x=2$	:	$3 \geqslant 7$	No
$x=10$	:	$11 \geqslant 7$	Yes
$x=20$	:	$21 \geqslant 7$	Yes

To solve an inequality → Find the values of x that make it true.

Solve $x+1 \geqslant 7$ → which values of x make this true

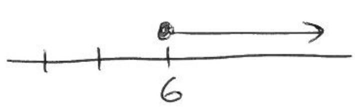
We can see once $x \geqslant 6$ then it is true

↗ Notice we have a whole range of values that could be solutions.

ie: infinitely many solns

So here $x+1 \geqslant 7$ has soln $x \geqslant 6$

ie:



we can also solve this algebraically

$$x+1 \geqslant 7$$
$$x \geqslant 7-1$$
$$x \geqslant 6$$

We rearrange like we would if there was an equal sign (to isolate x).

BUT There is 1 important rule

When $\times$ or $\div$ both sides of an inequality by a negative number, the inequality sign must be reversed.

eg: we know $-2 < 10$

mult by -1: $2 < -10$ ← NO

$\therefore$ need to reverse sign: $2 > -10$ ✓

eg. Solve $-2x < 4$

$$x > \frac{4}{-2}$$ ← Dividing by neg $\therefore$ Reverse ineq sign

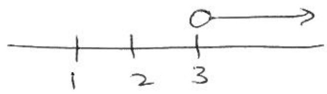
$$\therefore x > -2$$

Eg: Solve

1a) $2x+1 > 7$

$2x > 6$

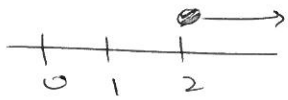
$x > 3$



b) $11 - 3x \leq 5$

$-3x \leq -6$

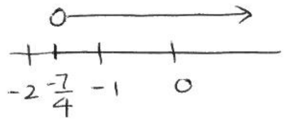
$x \geq 2$



c) $5 - 4x < 12$

$-4x < 7$

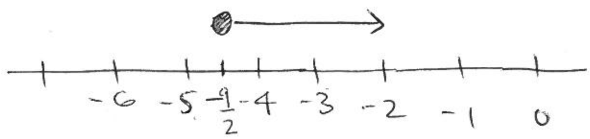
$x > -\frac{7}{4}$



d) $9 + 2x \geq 0$

$2x \geq -9$

$x \geq -\frac{9}{2}$

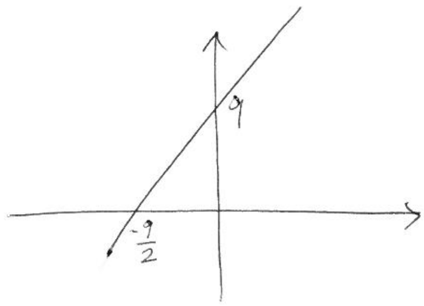


We can also see these solutions graphically.

Look at $9+2x \geq 0$

↗ recognise this

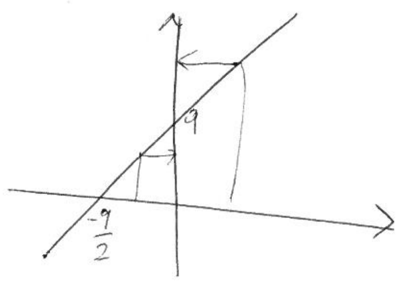
Consider the line $y = 9+2x$



For $9+2x \geq 0$ ← this says which x-values give us positive y-values.

From graph this corresponds to the part of the line above the x-axis

ie:



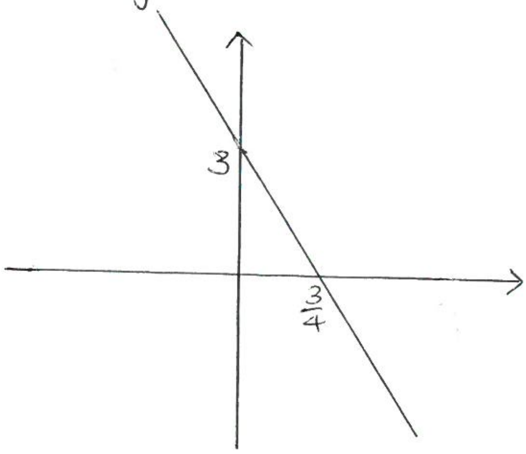
$\therefore$ Take $x \geq -\frac{9}{2}$

eg3) Solve $9-4x \leq 6$ by graphing.

$$9-4x \leq 6$$

ie: $3-4x \leq 0$

Sketch the line $y = 3-4x$



x-int → $y=0$

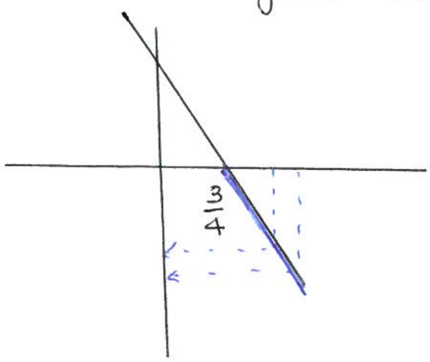
$3-4x=0$

$-4x=-3$

$x=\frac{3}{4}$

Since we want $3-4x \leq 0$

↗ choose part of line that gives neg y-values



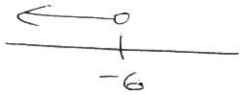
$\therefore \ x \geq \frac{3}{4}$

Solve:

3 a) $x+4 > 3x+16$

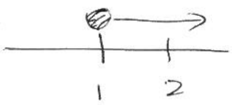
$-2x > 12$

$x < \frac{12}{-2}$

$x < -6$ ie:  ie: $(-\infty, -6)$

b) $3x+11 \leq 6x+8$

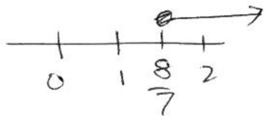
$-3x \leq -3$

$x \geq 1$  ie: $[1, \infty)$

c) $5y-6 \geq 2(1-y)$

ie: $5y-6 \geq 2-2y$

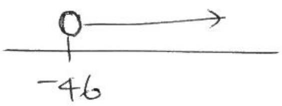
$7y \geq 8$

$y \geq \frac{8}{7}$  $[\frac{8}{7}, \infty)$

d) $3(y-5) - 4(y+6) < 7$

$3y - 15 - 4y - 24 < 7$

$-y - 39 < 7$

$-y < 46$  $(-46, \infty)$

$y > -46$

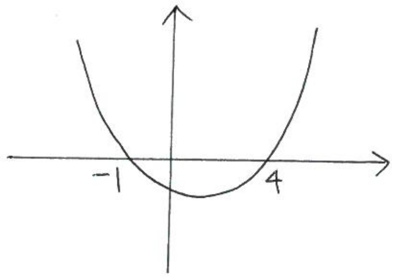
Quadratic Inequalities

- make use of the graphing technique:

eg4a) Solve $x^2 - 3x - 4 > 0$

Lets sketch $y = x^2 - 3x - 4$

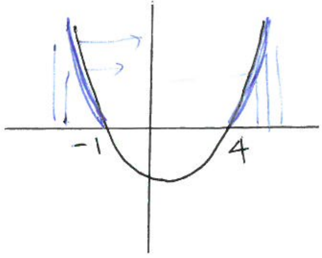
$= (x-4)(x+1)$



For $x^2 - 3x - 4 > 0$

want x values that make y pos.

$\therefore$ Take part of parabola above x axis (y pos)



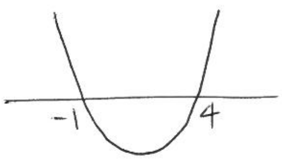
ie: $x < -1$ or $x > 4$

eg b) Solve $x^2 - 3x - 4 \geq 0$

Now we can include when its 0 as well

$\therefore x \leq -1, \; x \geq 4$

eg c) Solve $x^2 - 3x - 4 < 0$



Now take part of parabola below x axis.

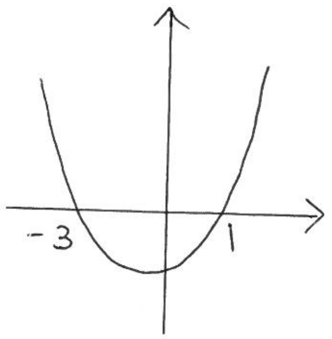
ie: $-1 < x < 4$

eg d) Solve $x^2 + 2x + 5 > 8$

First get everything on one side: $x^2 + 2x - 3 > 0$

Now look at $y = x^2 + 2x - 3$

$= (x+3)(x-1)$



Want pos y-values $\Rightarrow$ take part of parabola above x-axis

ie: $x < -3$, $x > 1$