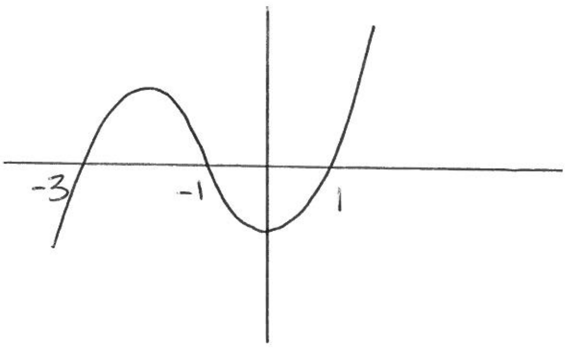
2d) Sketch $y = x^3 + 3x^2 - x - 3$

$= x^2(x+3) - (x+3)$

$= (x^2-1)(x+3)$

$= (x+1)(x-1)(x+3)$



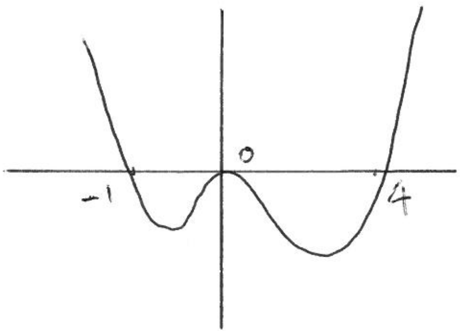

test:

$x = 2 : y = 3(1)(5) > 0$

e) Sketch $y = x^4 - 3x^3 - 4x^2$

$= x^2(x^2 - 3x - 4)$

$= x^2(x-4)(x+1)$



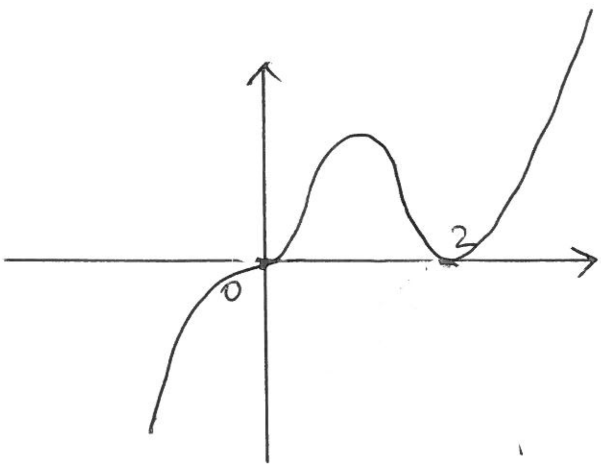

- deg 4
- facing up
- $x=0 \leftarrow$ double root
- $x = 4$, $x = -1$ $\leftarrow$ single roots

f) $y = x^5 - 4x^4 + 4x^3$

$= x^3(x^2 - 4x + 4)$

$= x^3(x-2)^2$

- deg 5
- $x=0 \leftarrow$ mult 3
- $x=2 \leftarrow$ mult 2 U
- test : $x = -1 : y = (-1)(3)^2 < 0$



# Operations on Polynomials

- were now going to look at polynomials algebraically

eg: $p(x) = x^3 + 2x^2 + 10$ $\qquad k(x) = 5x^2 - 1$

Addition + Subtraction - combine like terms

$$p(x) + k(x) = x^3 + 2x^2 + 10 + (5x^2 - 1)$$
$$= x^3 + 7x^2 + 9$$

$$p(x) - k(x) = x^3 + 2x^2 + 10 - (5x^2 - 1)$$
$$= x^3 + 2x^2 + 10 - 5x^2 + 1$$
$$= x^3 - 3x^2 + 11$$

Multiplication - expand + simplify product

$$p(x)\,k(x) = (x^3 + 2x^2 + 10)(5x^2 - 1)$$
$$= 5x^5 - x^3 + 10x^4 - 2x^2 + 50x^2 - 10$$
$$= 5x^5 + 10x^4 - x^3 - 48x^2 - 10$$

Substitution - sub values of $x$ to find values of poly.

eg: $p(1) = 1^3 + 2(1)^2 + 10 \quad = 13$

$k(0) = -1$

- Equality

- We say 2 polynomials are equal if the coefficients of the corresponding powers of $x$ are equal.

Let $p(x) = ax^4 - 7x^2 + d$

$g(x) = 5x^4 + kx^2 - mx - 1$

For $p(x) = g(x)$ Find $a, k, m, d$.

$\therefore$ We have $ax^4 - 7x^2 + d = 5x^4 + kx^2 - mx - 1$

equating coefficients : $a = 5$

$k = -7$

$m = 0$

$d = -1$

Find $a$ and $b$ where $a(x-2) + b(x+3) = x$

we want to equate coefficients

$\therefore$ Expand + regroup.

$ax - 2a + bx + 3b = x$

$(a+b)x + (-2a+3b) = x$

$$\left.\begin{aligned} \therefore\ a+b &= 1 \quad -① \\ -2a+3b &= 0 \quad -② \end{aligned}\right\} \text{solve simultaneously}$$

①×2 $\quad 2a + 2b = 2 \ -③$

③+② $\quad 5b = 2 \quad \rightarrow b = 2/5$

$\therefore$ sub in ① : $a = 1 - b = 1 - 2/5 = 3/5$

- Division

First recall with numbers $59 \div 7$

we say
$$\begin{array}{r} 8 \leftarrow \text{quotient} \\ 7\,\overline{)\,59} \\ \underline{56} \\ 3 \leftarrow \text{Remainder} \end{array}$$

so $59 = 7 \times 8 + 3$

dividend, divisor, quotient, remainder

& we require remainder to be smaller than divisor

- we're going to do something similar with polys.

eg: Divide $3x^3 + 5x^2 - x + 4$ by $x+2$

$$\begin{array}{r} 3x^2 - x + 1 \\ x+2\,\overline{)\,3x^3 + 5x^2 - x + 4} \\ \underline{3x^3 + 6x^2 \phantom{- x + 4}} \\ -x^2 - x + 4 \\ \underline{-x^2 - 2x \phantom{+ 4}} \\ x + 4 \\ \underline{x + 2} \\ 2 \end{array}$$

So $3x^3 + 5x^2 - x + 4$ divided by $x+2$ gives

quotient $= 3x^2 - x + 1$

remainder $= 2$

ie: $3x^3 + 5x^2 - x + 4 = (x+2)(3x^2 - x + 1) + 2$

dividend, divisor, quotient, remainder

eg Divide $x^3 + 4x^2 - 3x - 7$ by $x - 1$

$$
\begin{array}{r}
x^2 + 5x + 2 \\
x - 1 \overline{\smash{\big)}\, x^3 + 4x^2 - 3x - 7} \\
\underline{x^3 - x^2 \phantom{- 3x - 7}} \\
5x^2 - 3x \phantom{- 7} \\
\underline{5x^2 - 5x \phantom{- 7}} \\
2x - 7 \\
\underline{2x - 2} \\
-5
\end{array}
$$

$$\therefore x^3 + 4x^2 - 3x - 7 = (x-1)(\underbrace{x^2 + 5x + 2}_{\text{quotient}}) \underbrace{- 5}_{\text{remainder}}$$

Formally
- $p(x)$, $f(x)$ - polys $\deg p > \deg f$
- dividing $p(x)$ by $f(x)$ gives

$$p(x) = f(x)q(x) + r(x)$$

where $q(x)$ = quotient
$r(x)$ = remainder and $\deg r < \deg f$
or $r = 0$

says remainder is smaller than divisor.

or it fully divides into poly.