Yesterday

Polynomials eg: $p(x) = x^3 + 3x^2 - x - 3$

Deg = 3
Leading term = $x^3$
Const term = $-3$

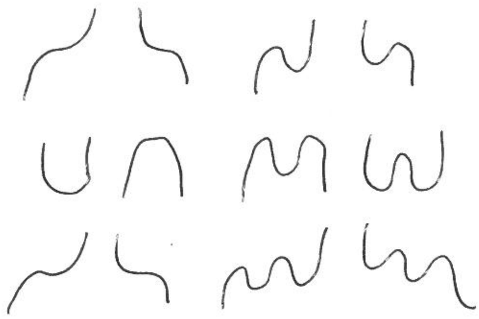
Lines - Deg 1
Quadratics - Deg 2
Cubics - Deg 3

* Odd Deg → start + end on opp sides

* Even Deg → start + end on same sides

Deg → tells us end behaviour
→ Deg − 1 = max n° of bumps
→ Deg = max n° of changes of direction

$x^3$
$x^4$
$x^5$



Polys - smooth + continuous
(Dom = $\mathbb{R}$)

eg Start by sketching quadratics :

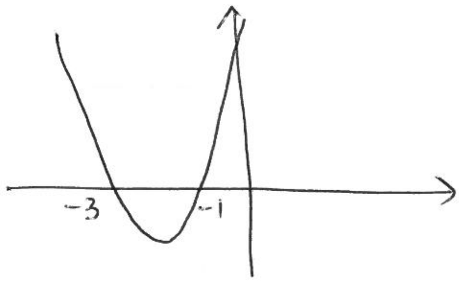
$y = x^2 + 4x + 3$

$x$ int $\rightarrow y = 0$

$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x = -3, -1$




$\leftarrow$ 2 $x$-ints

2 roots

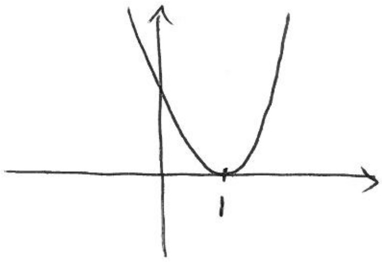
$y = x^2 - 2x + 1$

$= (x-1)^2$

$\therefore x$ int $\rightarrow y = 0$

$(x-1)^2 = 0$

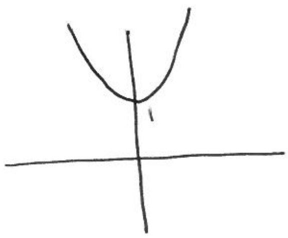
$x = 1$




$\leftarrow$ 1 $x$-int

- double root
- this root has multiplicity 2

$y = x^2 + 1$




- vertical shift
- no $x$-ints
- cant be written in factorised form

$\leftarrow$ 0 $x$ ints

Remember $\Delta = b^2 - 4ac$ will also give us this info

· Things to notice :

① Notice eqn + n° of roots.

$y = (x+3)(x+1)$     2 roots

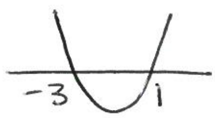
$y = (x+1)^2$     1 root

$y = x^2 + 1$     0 roots.

* If you can factorise, each factor will give a root.

② Look at $p(x) = x^2 + 4x + 3$

$= (x+3)(x+1)$




We say $x = -1$ is a solution to $p(x) = 0$

$x = -1$ " " root of the poly $p(x)$

$x = -1$ " " zero " " " eqn $p(x) = 0$

$x = -1$ " " x-intercept of $p(x)$.

* roots / solutions to $p(x) = 0$ give x-intercepts

* Learn to read off roots     $(x+3)(x+1) = 0$

↗ should be able to solve this in your head.

* A poly can't have more roots than its degree (ie: deg tells us max n° of roots)

③ Look at $y = (x-1)^2$ ← 1 root

. we say its a double root

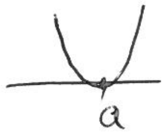
. ie: there is 1 repeated factor

. this root has multiplicity 2.

* Suppose $y = (x-a)^m$

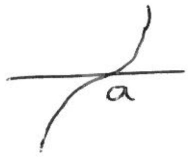
- $a$ is a root of multiplicity $m$

· If $m$ = even





$m$ = odd

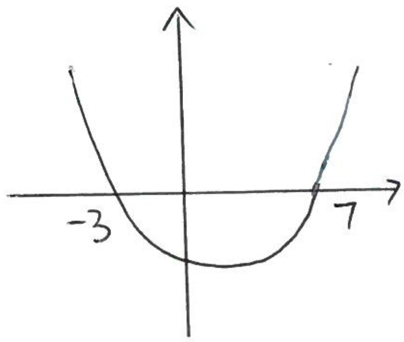




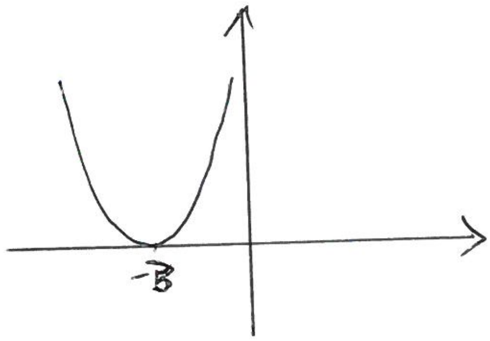
$(m > 1)$

eg: Find the zeros of the following quadratics + sketch their graphs.

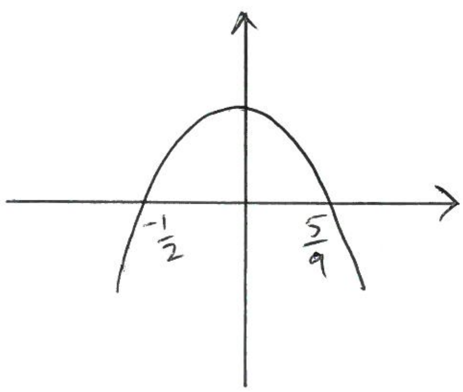
a) $y = (x-7)(x+3)$ ← parabola facing up
$x$ int at 7 and $-3$
(2 zeros)




b) $y = (x+3)^2$ ← double root at $x = -3$




c) $y = -3(9x-5)(2x+1)$ ← facing down
2 zeros when
$9x - 5 = 0 \rightarrow x = 5/9$
$2x + 1 = 0 \rightarrow x = -1/2$

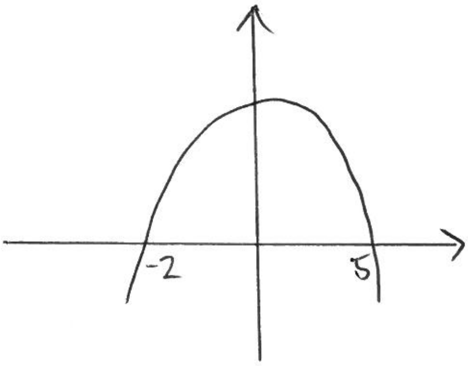


d) $y = 10 + 3x - x^2$

$= -(x^2 - 3x - 10)$

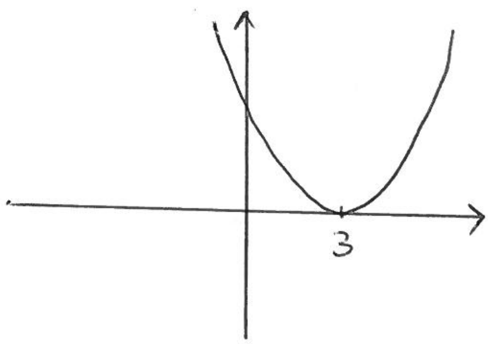
$= -(x-5)(x+2)$

zeros at $x = 5, -2$




e) $y = x^2 - 6x + 9$

$= (x-3)^2$



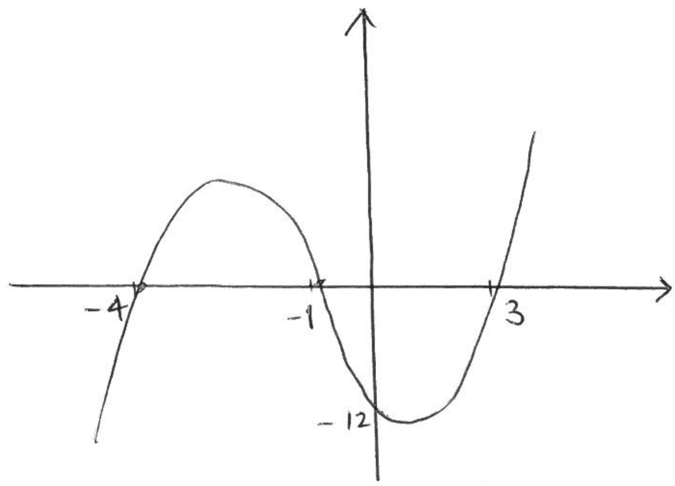
# Sketching cubics and higher powers

- zeros → tell us intercepts + multiplicity of roots
- remember end behaviours
- test points if needed.

eg a) Sketch $y = (x+1)(x-3)(x+4)$

↖ cubic $y = x^3 + \cdots$
with 3 zeros at $-1, 3, -4$

∴ Either 

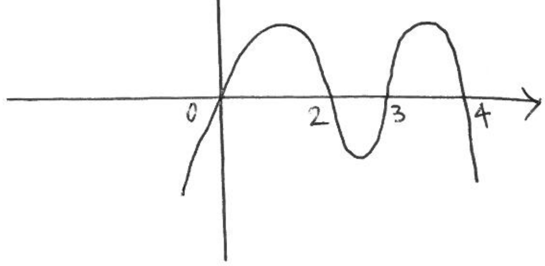



↖ test point to see which one.

$x = 0$ : $y = 1(-3)(4)$
$= -12$

b) Sketch $y = -2x(x-2)(x-3)(x-4)$

← . degree = 4
. facing down
. 4 zeros at
0, 2, 3, 4

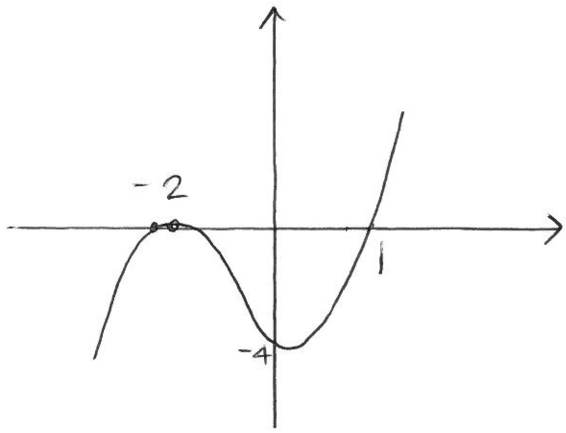


Extra Questions:

1) Sketch $y = (x+2)^2(x-1)$

- degree = 3
- double root at $x = -2$

↳ ∴ its an intercept and turning point




test point: $x=0$

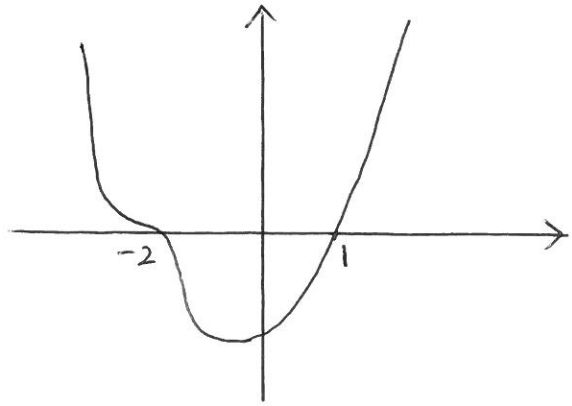
$y = (2)^2(-1) = -4$

2) Sketch $y = (x+2)^3(x-1)$

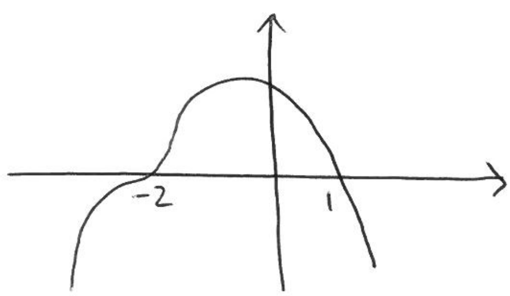
- degree 4
- root at $x = -2$ of multiplicity 3

ie:

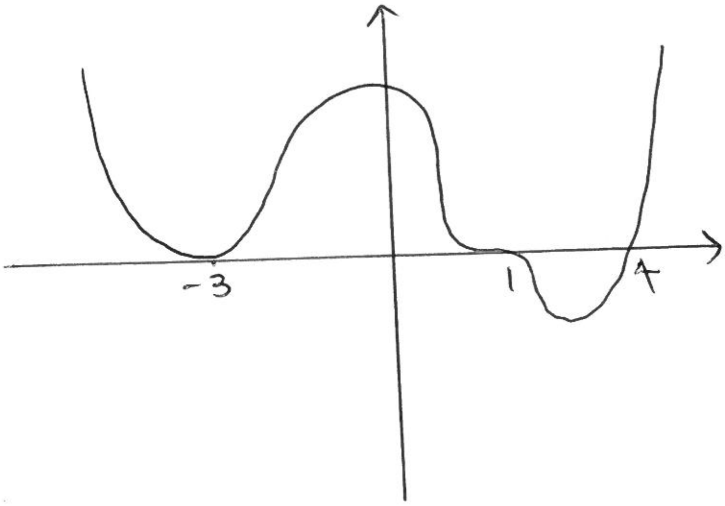
- facing up




3) Sketch $y = (x+2)^3(1-x)$



2c) Sketch $y = (x-4)(x+3)^2(x-1)^3$




· deg 6 - facing up

· roots at
$x=4$
$x=-3$ ← mult 2
$x=+1$ ← mult 3