Polynomials

Defn: A polynomial is an expression of the form

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \quad (a_n \neq 0).$$

$a_0, a_1, \cdots, a_n$ = constants, called coefficients

a_0 = term without x, called constant term

x = variable

eg 1) $4x^3 + x^2 + 7x + 3$ is a poly

- coeff of x^3 is 4
- coeff of x^2 is 1
- coeff of x is 7
- constant term is 3

2) $10 - 3x^4 + x^6$ is a poly

- coeff of x^4 is -3
- constant term is 10

3) $8x - 2x^3$ is a poly

coeff of x^3 is -2
coeff of x^2 is 0 $\leftarrow$ ie: $-2x^3 + 0x^2 + 8x + 0$
constant term is 0

4) $x^3 + 2x^2 + x^{1/2} + 3$ is NOT a poly ← problem

5) $\sqrt{x} + x^2$ is NOT a poly

6) $\frac{1}{x^6} - x + 1$ is NOT a poly

7) $x^3 + x^{-2} + x + 1$ is NOT a poly

- Degree = highest power of x that appears in poly.

From previous egs

1) deg = 3
2) deg = 6
3) deg = 3

- Leading term = term that contains highest power of x
- Leading coeff = coeff of leading term

eg:

$$3x^5 + \frac{1}{2}x^4 - 7x^3 - x^2 + 2x - 5$$

Degree = 5

Leading term = $3x^5$

Leading coeff = 3

Constant term = -5

From previous eg.

1) $4x^3 + x^2 + 7x + 3$ has leading term $= 4x^3$
leading coeff $= 4$

2) $10 - 3x^4 + x^6$ has leading term $= x^6$
leading coeff $= 1$

3) $8x - 2x^3$ has leading term $= -2x^3$
leading coeff $= -2$

We usually use function notation when working with polynomials.

eg: $p(x) = 4x^3 + x^2 + 7x + 3$ $\quad k(x) = 8x - 2x^3$

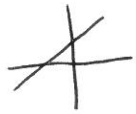
same as $y = 4x^3 + x^2 + 7x + 3$

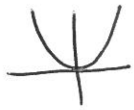
- Polynomials are functions.

Polynomials we know :

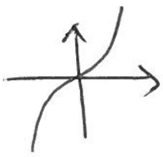
- We have already met some polynomials.

- Lines are polynomials $y = mx + b$ (or $p(x) = a_1 x + a_0$)
(deg 1)



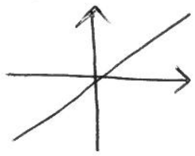
- Quadratics $y = ax^2 + bx + c$ are deg 2 polynomials 

- The cubic $y = x^3$ is a deg 3 poly.

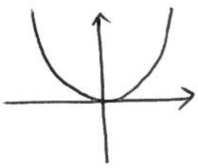


Graphing Polynomials

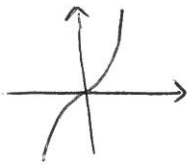
Basic polys:



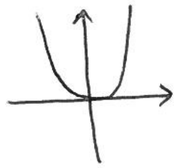
$y = x$



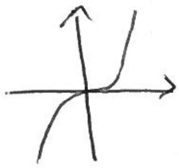
$y = x^2$



$y = x^3$



$y = x^4$



$y = x^5$

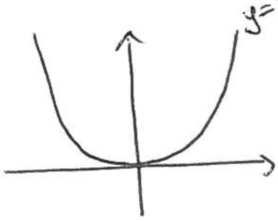
Notice : • odd degree – start + end on opposite sides
• even degree – " " " " same "

ie:

$y = x^2$

as $x \to \infty$
$y \to +\infty$

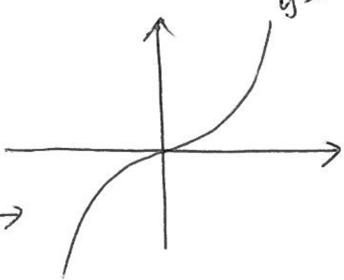
as $x \to -\infty$
$y \to +\infty$



← End behaviour of polys

$y = x^3$

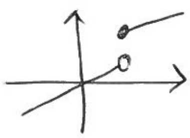
as $x \to +\infty$
$y \to +\infty$



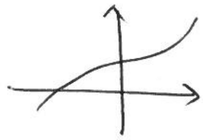
as $x \to -\infty$
$y \to -\infty$

- Graphs of polynomials are continuous

all joined up (No breaks or holes)



Not continuous

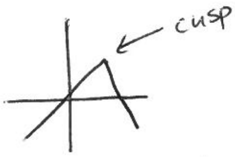


continuous

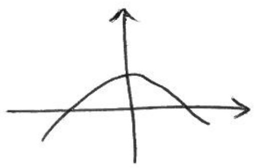
This is obvious since their domain $= \mathbb{R}$

ie: they exist for all $\mathbb{R}$

- Graphs of polynomials are smooth



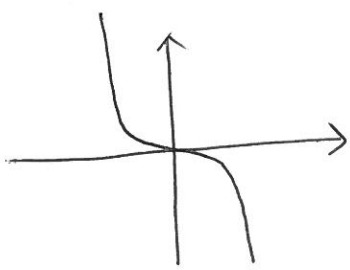
Not smooth



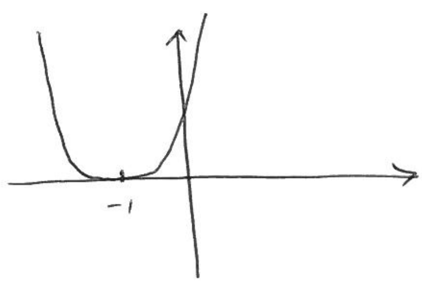
Smooth

- Knowing all of this we can sketch graphs of polys which are transformations of the basic ones.

eg a) $y = -x^5$



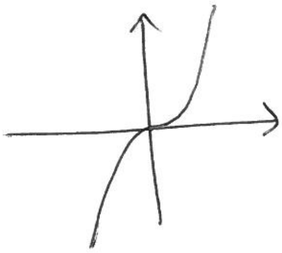
b) $y = (x+1)^4$



horizontal shift

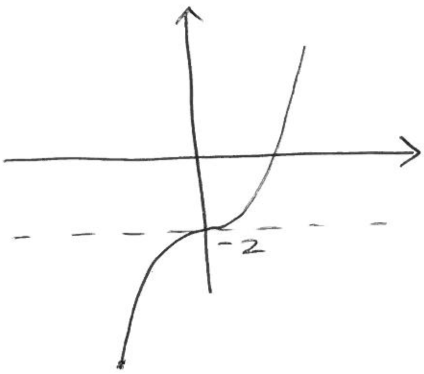
c) $y = x^9 - 2$

Recall $y = x^9$



odd degree so start + end on different sides

So $y = x^9 - 2$



vertical shift

- But what about more complex polynomials

eg: $y = x^3 + 3x^2 - x - 3$ ← This is not a transformation of x^3

- We know end behaviour
  - as x gets larger, x^3 dominates
  - ∴ end behaviour determined by leading term

- The only other difference is there could be an extra bump or wiggle.

So $y = x^3 + \cdots$ could look like

or

When sketching :

Leading term — Basic graph

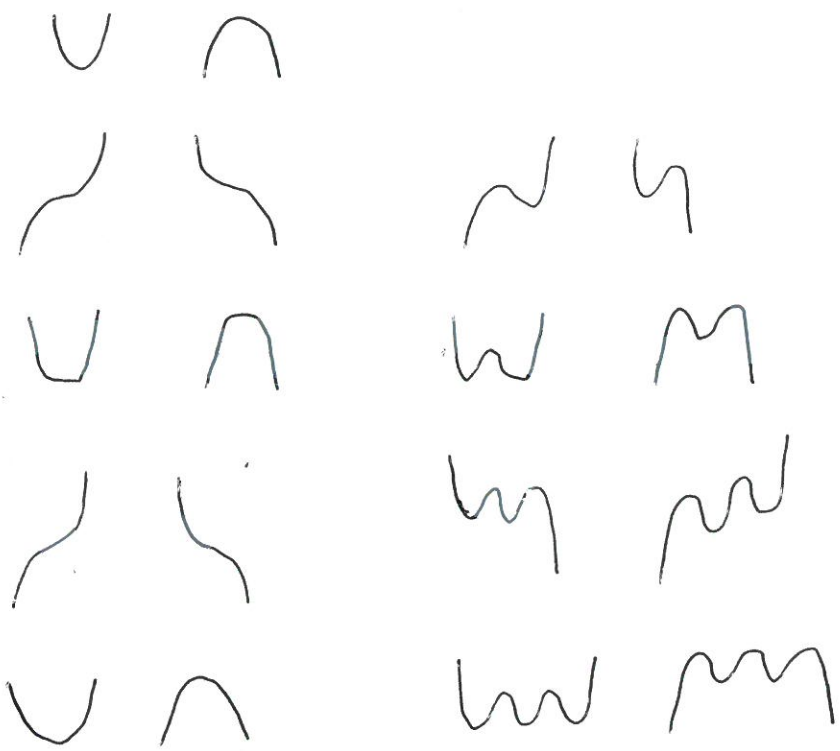
x^2

x^3

x^4

x^5

x^6



- Degree - tells us behaviour ← what happens as $x \to \pm\infty$
  - tells us max number of bumps (1 less than deg)
  - " " " " " changes of direction.

- Since polys are smooth + continuous, if we know the x-intercepts, then we will know how to draw them