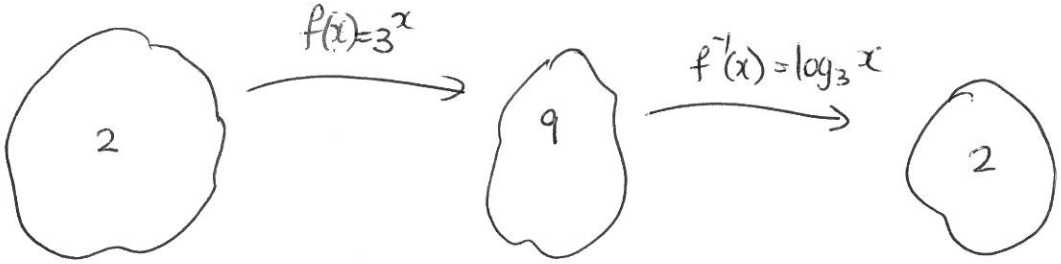
Inverse Relationship

Logs + Exp are inverses.

eg. If $f(x) = 3^x$ then $f^{-1}(x) = \log_3 x$

eg:



$\log_3 9 = \log_3 3^2 = 2$.

Similarly $f(x) = e^x$ then $f^{-1}(x) = \log_e x$.

In general $f(x) = b^x$ then $f^{-1}(x) = \log_b x$

We can see the inverse relationship by looking at the definition of inverse

ie: $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$.

ie: $(f \circ f^{-1})(x) = f(f^{-1}(x)) = f(\log_b x) = b^{\log_b x} = x$

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(b^x) = \log_b b^x = x$

↑ Recall these 2 properties of logs.

eg) Show $f(x) = 5e^x$ and $g(x) = \log\left(\frac{x}{5}\right)$ are inverses.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f\left(\log \tfrac{x}{5}\right) \\ &= 5e^{\log(x/5)} \\ &= 5\left(\tfrac{x}{5}\right) \\ &= x\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(5e^x) \\ &= \log\left(\tfrac{5e^x}{5}\right) \\ &= \log e^x \\ &= x\end{aligned}$$

$\therefore$ $f(x)$ and $g(x)$ are inverses and we can say $f^{-1}(x) = \log\left(\frac{x}{5}\right)$

eg Find the inverse of $f(x) = e^{x+3}$ and sketch both functions

Remember how we found inverses.

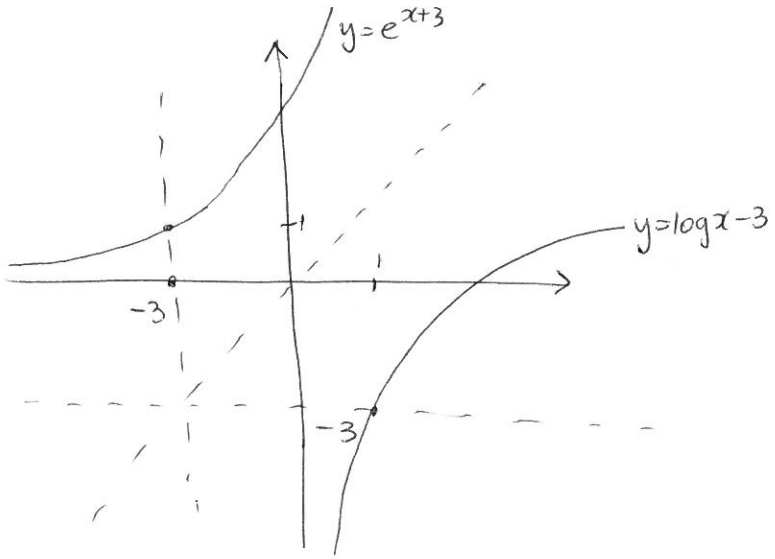
Let $y = e^{x+3}$

Swap roles of x and y $\quad x = e^{y+3}$

rearrange $\quad \log x = \log e^{y+3}$

ie: $\quad \log x = y + 3$

$\therefore y = \log x - 3$ $\qquad$ $\therefore f^{-1}(x) = \log x - 3$



$f(x) : \mathbb{R} \to (0, \infty)$ $\quad$ and $\quad f^{-1}(x) : (0, \infty) \to \mathbb{R}$

Solving equations with powers

Now that we know about the relationship between logs and exponentials we can solve equations where x is in the power.

eg Solve $e^{3x} = 15$

$$3x = \log 15$$
$$x = \frac{1}{3}\log 15$$

Solve $12e^{3x} = 15$

$$e^{3x} = \frac{15}{12} = \frac{5}{4}$$

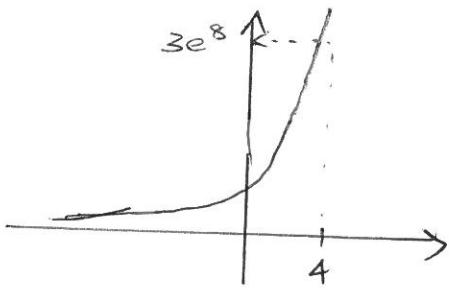
$$\therefore 3x = \log\left(\frac{5}{4}\right)$$
$$x = \frac{1}{3}\log\left(\frac{5}{4}\right)$$

Solve $512 - 3e^{-7x} = 210$

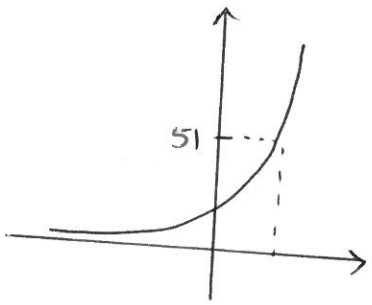
$$-3e^{-7x} = -302$$
$$e^{-7x} = \frac{302}{3}$$
$$\ln e^{-7x} = \ln\left(\frac{302}{3}\right)$$
$$-7x = \ln\left(\frac{302}{3}\right)$$
$$x = -\frac{1}{7}\ln\left(\frac{302}{3}\right)$$

$f(x) = 3e^{2x}$

a) when $x = 4$ $\quad f(x) = 3e^{2(4)} = 3e^8 \approx 8942$



b) Now we want x when $f(x) = 51$



$\therefore 3e^{2x} = 51$

$e^{2x} = \frac{51}{3}$

$e^{2x} = 17$

$\log e^{2x} = \log 17$

$\therefore 2x = \log 17$

$x = \frac{1}{2} \log 17 \approx 1.42$

c) want x when $f(x) = 0.4$

$\therefore 3e^{2x} = 0.4$

$e^{2x} = \frac{0.4}{3}$

ie: $e^{2x} = \frac{2}{15}$

$\therefore \ln e^{2x} = \ln\left(\frac{2}{15}\right)$

$2x = \ln\left(\frac{2}{15}\right)$

$x = \frac{1}{2} \ln\left(\frac{2}{15}\right)$